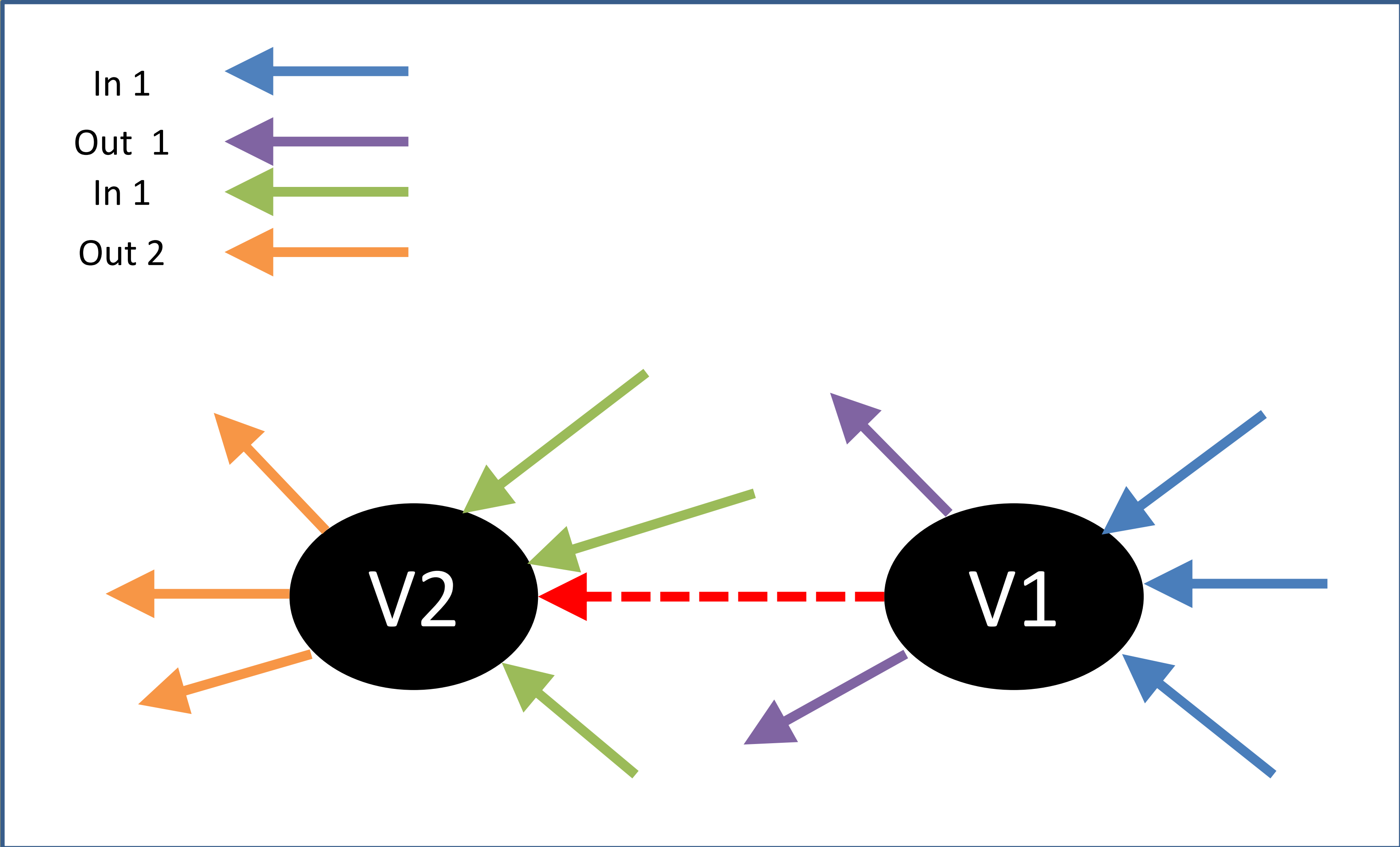
Directional triadic closure and edge deletion mechanism induce asymmetry in directed edge properties.

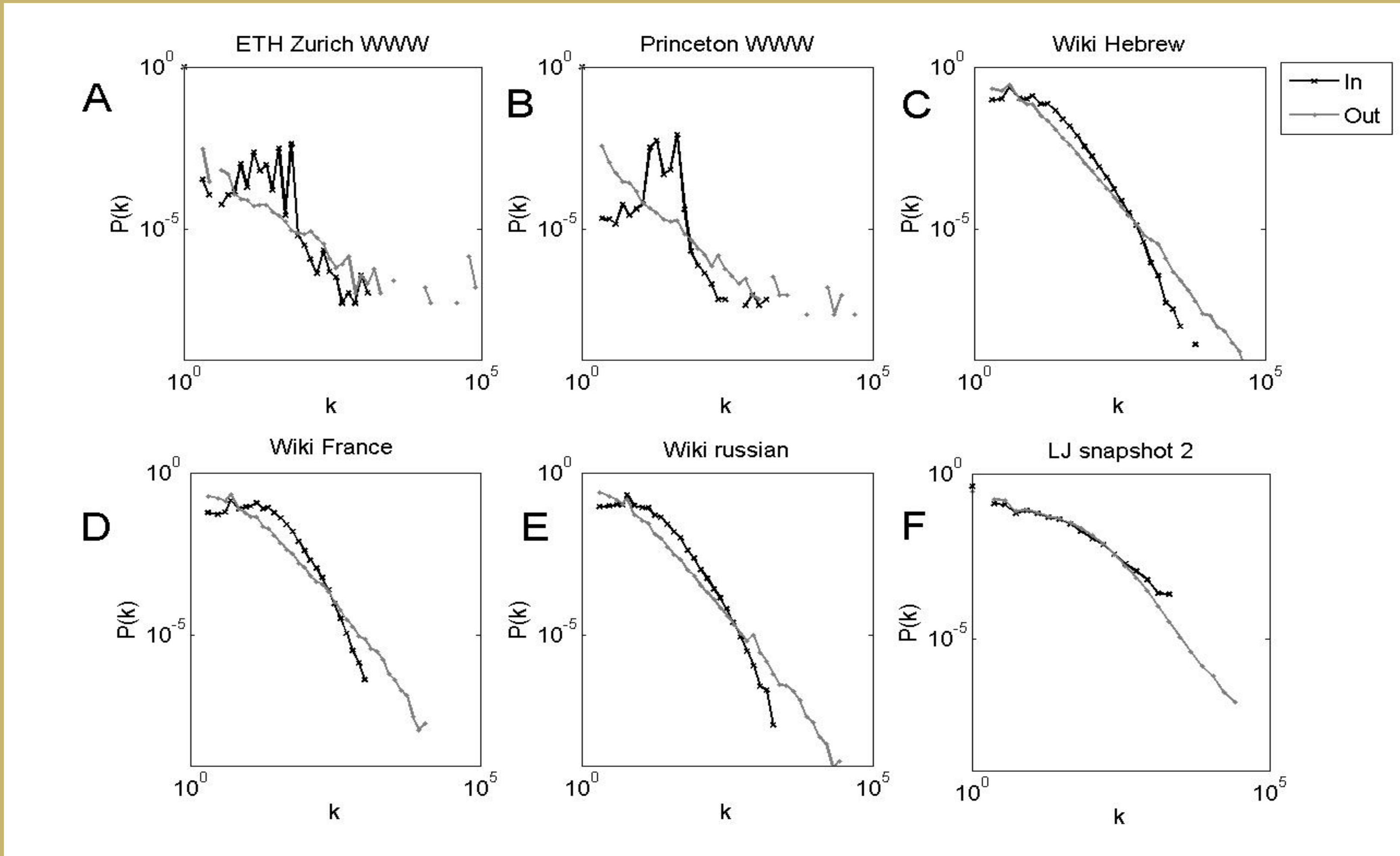
Directional Networks

- Two of the most consistent features of real world networks are the scale free degree distributions and the high clustering coefficients.
- In directed networks, the in and out clustering coefficients differ one from each other. Similarly the in and out degree often have different distributions, the frequency of different triangles is not uniform, and directed clustering coefficients is different.
- Most network generation models do not incorporate the differences between in and out degree properties.

Directional Networks



Different between in and out degree properties in real world networks



Different between clustering coefficient and directional degree correlation in real world networks

Network	Direct CC	Inverse CC	Correlation between in/out degree
ETH Zurich WWW	0.0847	0.3865	0.0034
Princeton WWW	0.0672	0.3173	0.0016
Wiki Russian	0.094	0.128	0.2566
Wiki Hebrew	0.1036	0.1373	0.362
Wiki France	0.1228	0.1397	0.2943
LJ	0.1416	0.1372	0.2867

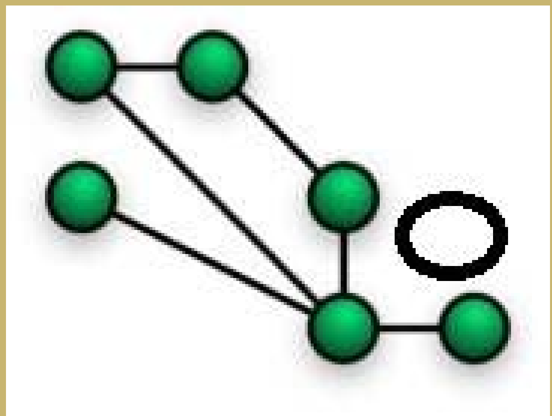
Triadic closure

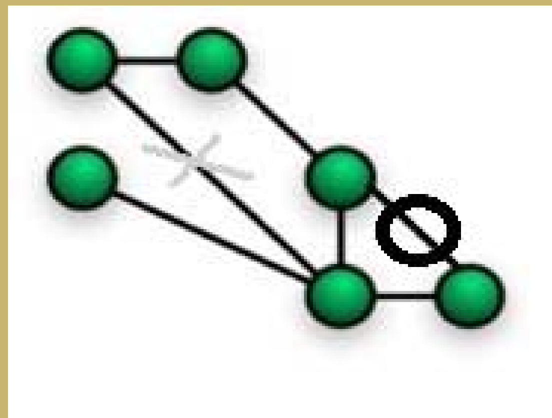
- Triadic closure has been suggested as a socially plausible mechanism capable to generate undirected networks with the realistic properties. In the basic version of this model, a random node is selected and an edge is created between two of its first neighbors. Extensions of this basic model includes (among others):
  - Random walkers,
  - Involvement of second or higher order neighbors,
  - Combinations with preferential attachment,
  - Creation of new nodes
  - Random edge creation

- This mimic the basic social phenomena of people who meet new friends through mutual acquaintance.

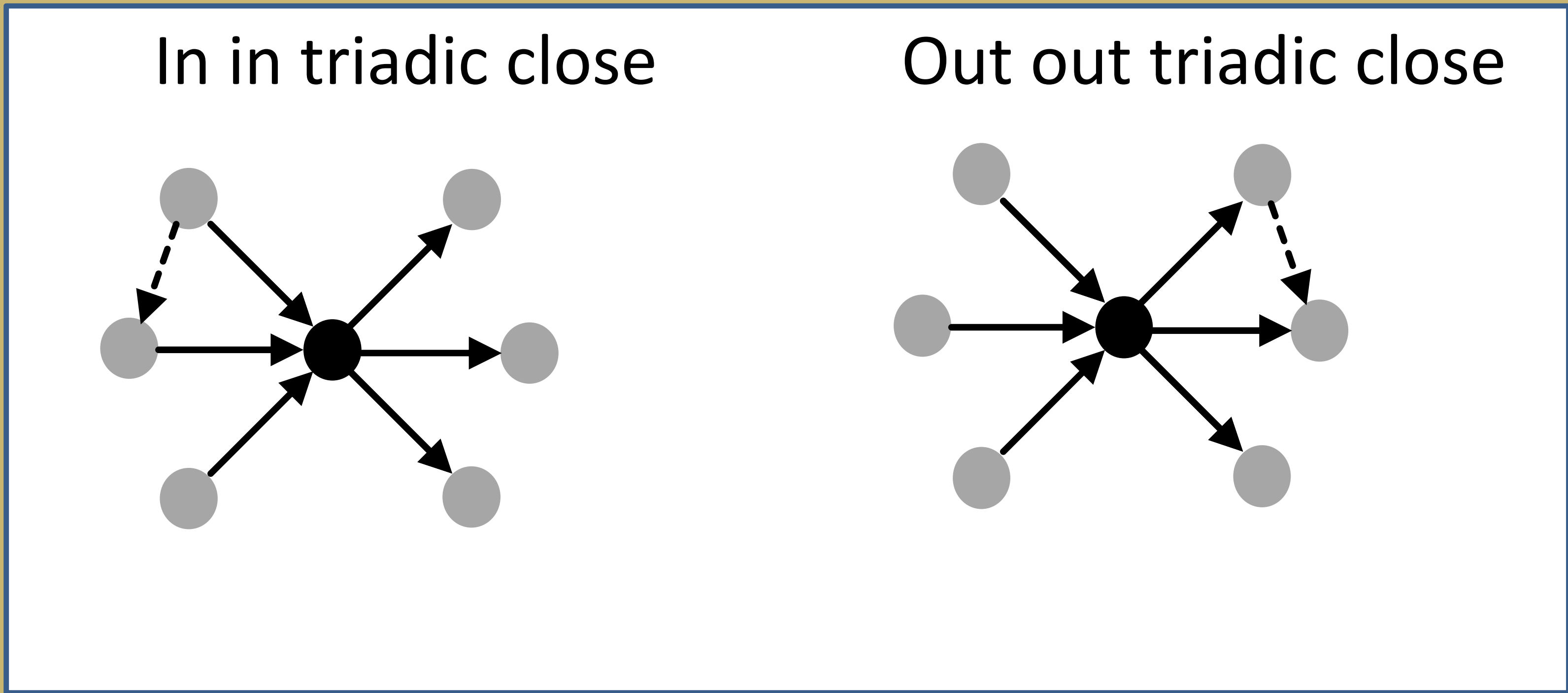
Edges deletion

- Edge deletion properties have received little attention in network research.
- Even when edge deletion processes were considered, their goal was to maintain the number of edges in the network by removing edges or entire nodes randomly.
- Our recent work shows that complex edge deletion mechanisms are indeed observed in real world networks Brot, H., et al., *Edge removal balances preferential attachment and triad closing.* Physical Review E, 2013. **88**(4): p. 042815.





Directional triadic closure with random edge deletion-connecting between similar direction



Directional triadic closure with random edge deletion-connecting between similar direction

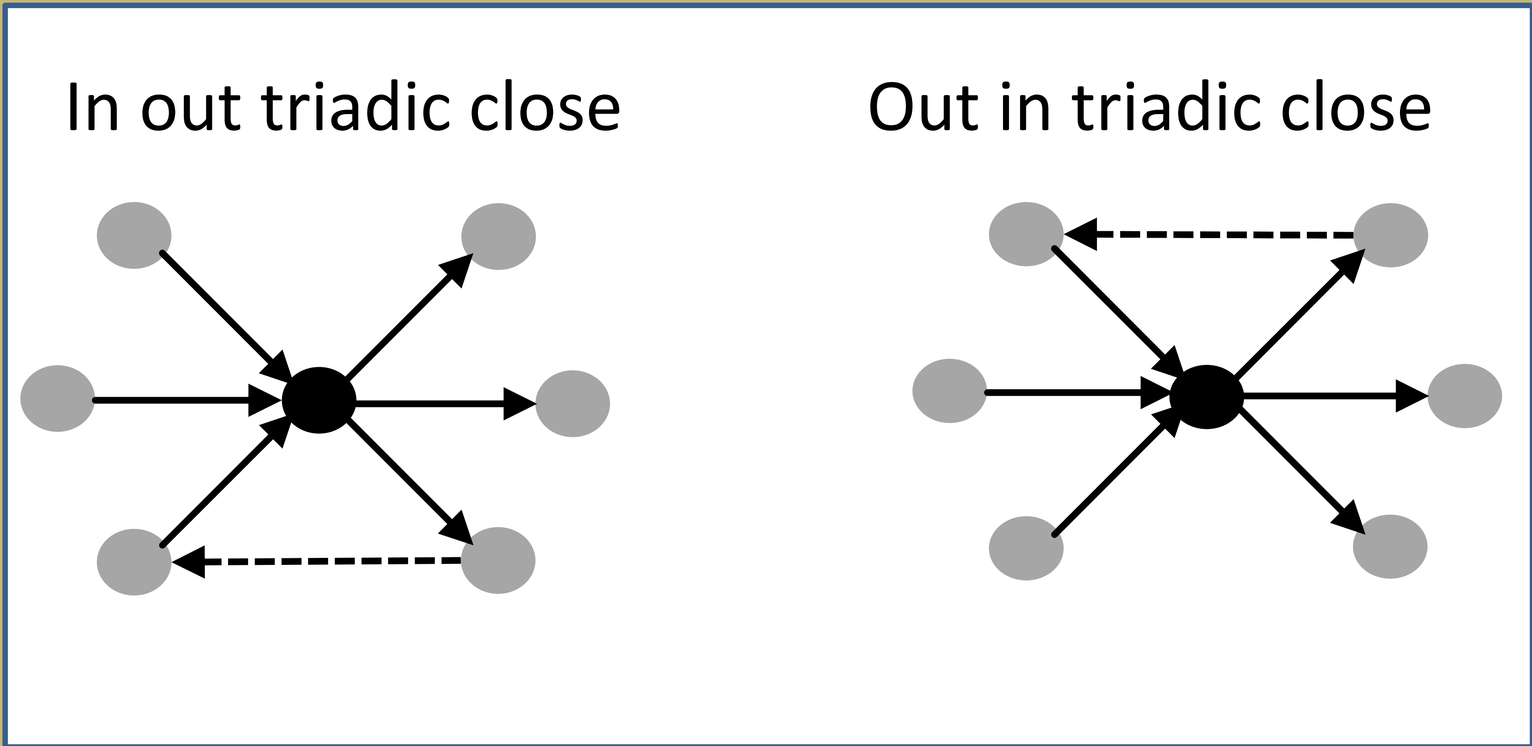
Generic birth death process:

$$p(k_{in} \to k_{in}+1 \mid k_{out}) = \frac{k_{out}}{K};\ p(k_{in} \to k_{in}-1 \mid k_{out}) = \frac{k_{in}}{K}$$
$$p(k_{out} \to k_{out}+1 \mid k_{in}) = \frac{k_{out}}{K};\ p(k_{out} \to k_{out}-1 \mid k_{in}) = \frac{k_{out}}{K}$$

K is the average in or out degree. The resulting Fokker-Plank equations are:

$$\frac{\partial p(k_{in} \mid k_{out})}{\partial t} = -\frac{\partial p(k_{in})\left(k_{out} - k_{in}\right)}{K \partial k_{in}} + \frac{1}{2}\frac{\partial^2 p(k_{in})\left(k_{in}\right)}{K \partial {k_{in}}^2}$$
$$\frac{\partial p(k_{out} \mid k_{in})}{\partial t} = \frac{1}{2}\frac{\partial^2 p(k_{out})\left(k_{out}\right)}{K \partial {k_{out}}^2}$$

Directional triadic closure with random edge deletion-connecting between opposite directions



Directional triadic closure with random edge deletion-connecting between different direction

Generic birth death process:

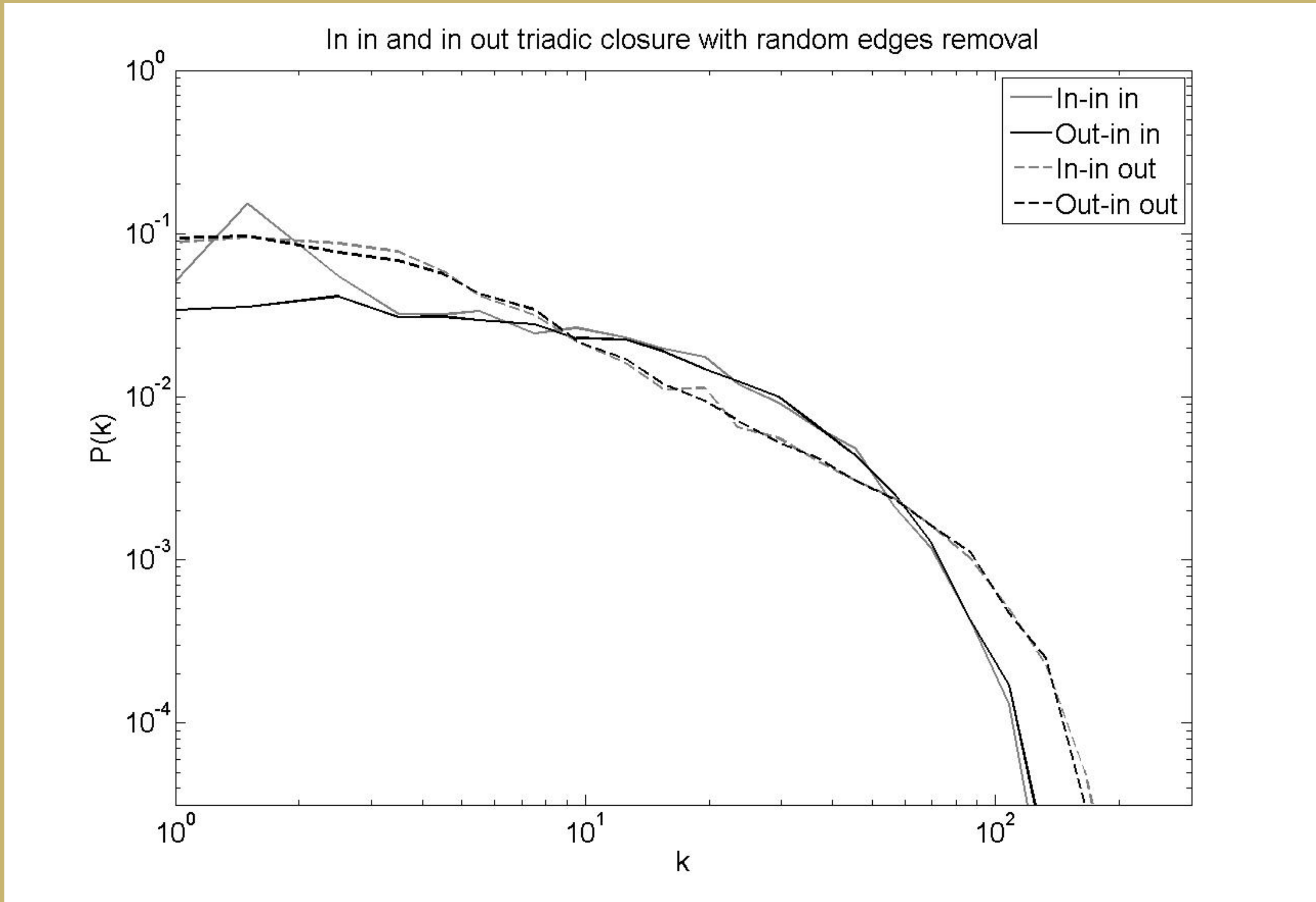
$$p(k_{in} \to k_{in}+1 \mid k_{out}) = \frac{1}{2}\cdot\left(\frac{k_{in}}{K}+\frac{k_{out}}{K}\right);\ p(k_{in} \to k_{in}-1 \mid k_{out}) = \frac{k_{in}}{K}$$

$$p(k_{out} \to k_{out}+1 \mid k_{in}) = \frac{1}{2}\cdot\left(\frac{k_{in}}{K}+\frac{k_{out}}{K}\right);\ p(k_{out} \to k_{out}-1 \mid k_{in}) = \frac{k_{out}}{K}$$

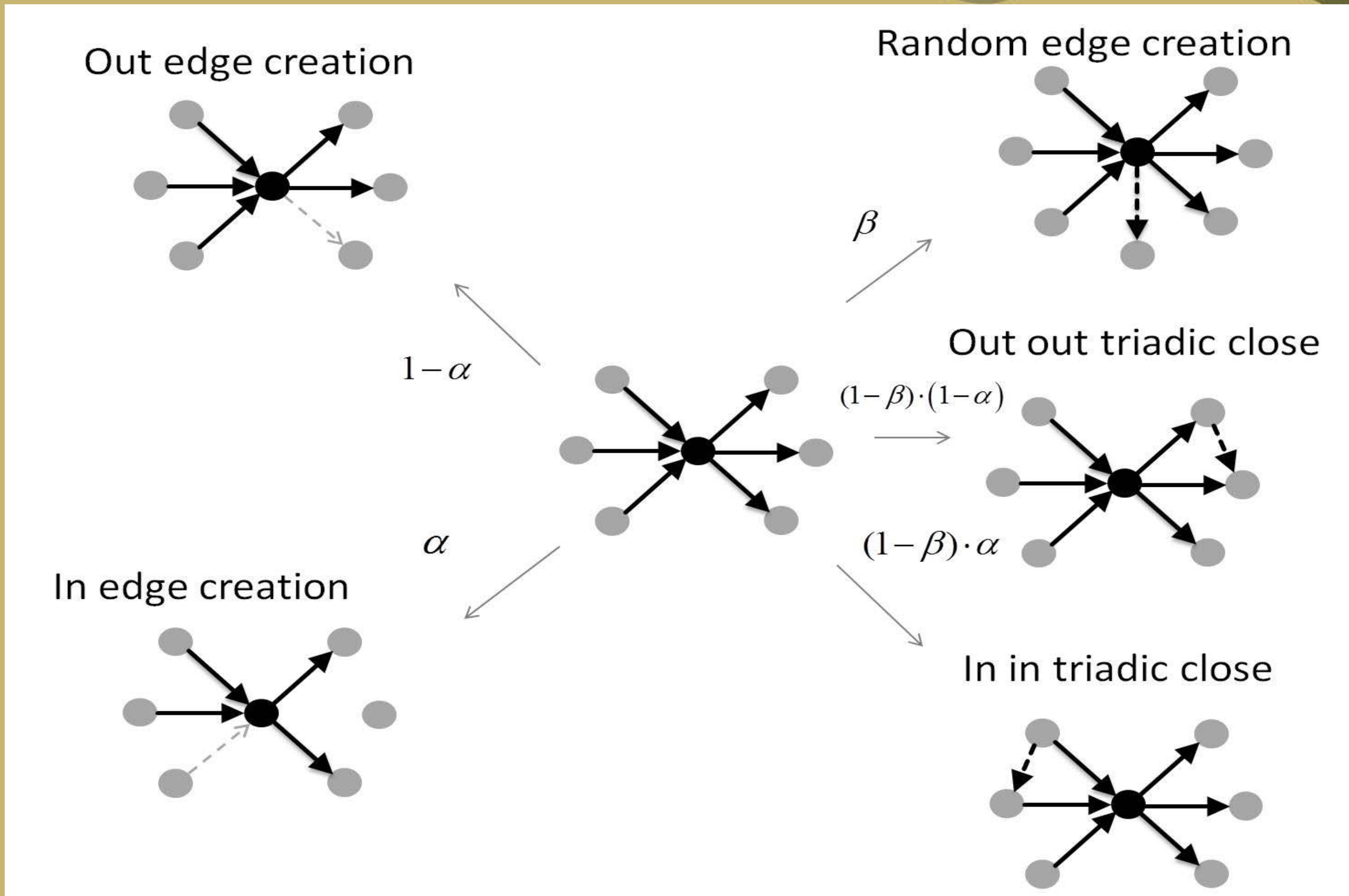
Both models results with the same degree distribution without any change between in to out degree distribution.

The correlation between the in-degree and the out-degree in this first model is almost 1 (0.97), while in the second model creates an uncorrelated degree (-0.0287)

Degree distribution for directed triadic closure with random edge removal



Suggested model



Suggested model –Complete edges removal addition probabilities

In degree addition:

$$p(k_{in} \mid k_{out} \rightarrow k_{in}+1) =$$
$$\frac{\beta\alpha}{K}\left[k_{in}-k_{out}\right]+\beta\frac{k_{out}}{K}-\beta\frac{k_{out}}{K}p\left(k_{in}<2\right)-\alpha\beta\frac{k_{in}}{K}p\left(k_{out}<2\right)+1-\beta$$
$$+\beta p\left(k_{in}<2\right)+\alpha\beta p\left(k_{out}<2\right)$$

In degree removal:

$$p(k_{in} \mid k_{out} \rightarrow k_{in}-1)=1-\alpha+\alpha\frac{k_{in}}{K}$$

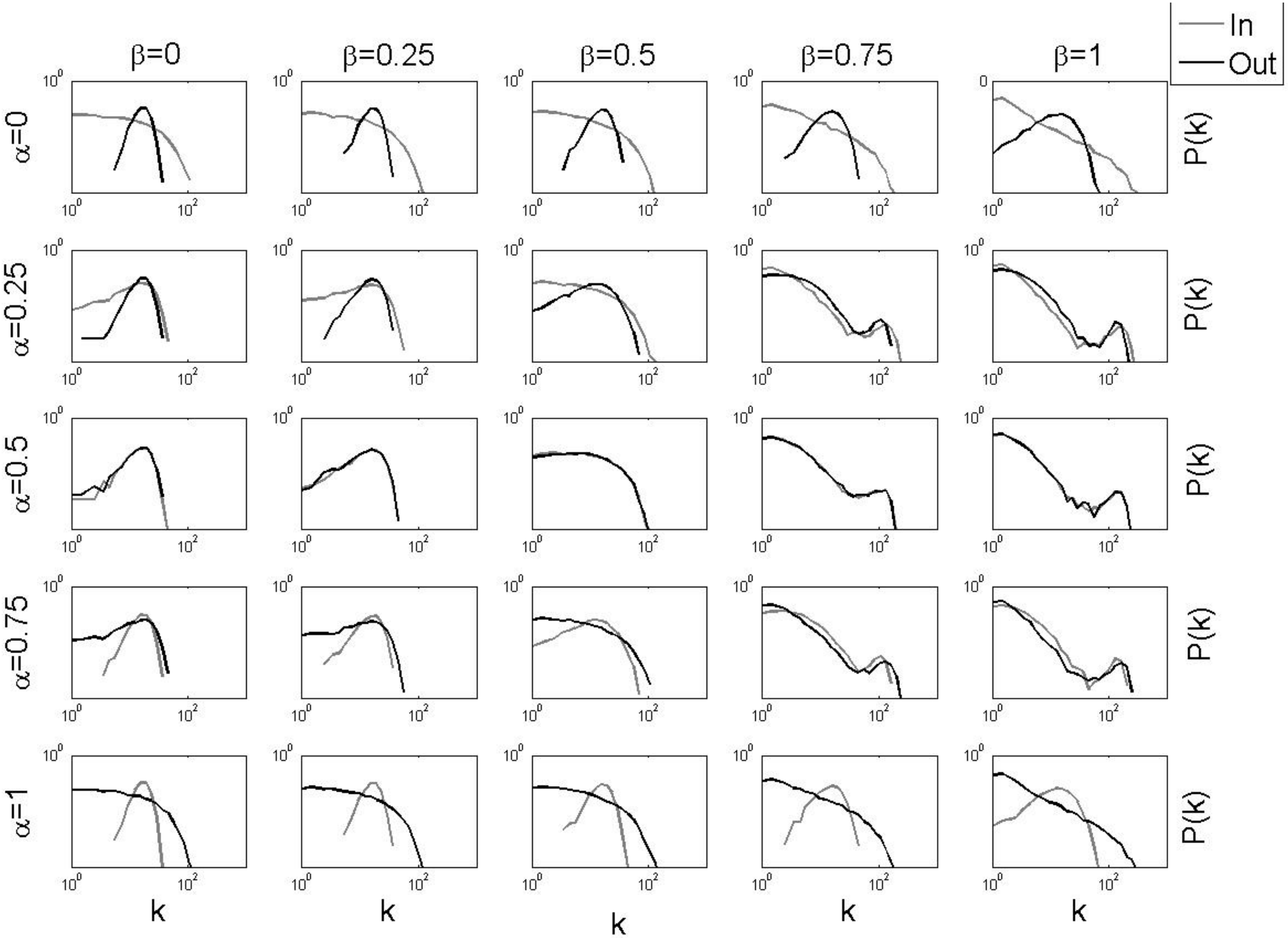
Out degree addition:

$$p(k_{out} \mid k_{in} \rightarrow k_{out}+1)=\beta\alpha\frac{k_{in}}{K}p\left(k_{out}>1\right)+\left(1-\alpha\right)\beta\frac{k_{out}}{K}p\left(k_{in}>1\right)+1-\beta$$
$$+\beta\alpha p\left(k_{out}<2\right)+\left(1-\alpha\right)\beta p\left(k_{in}<2\right)$$

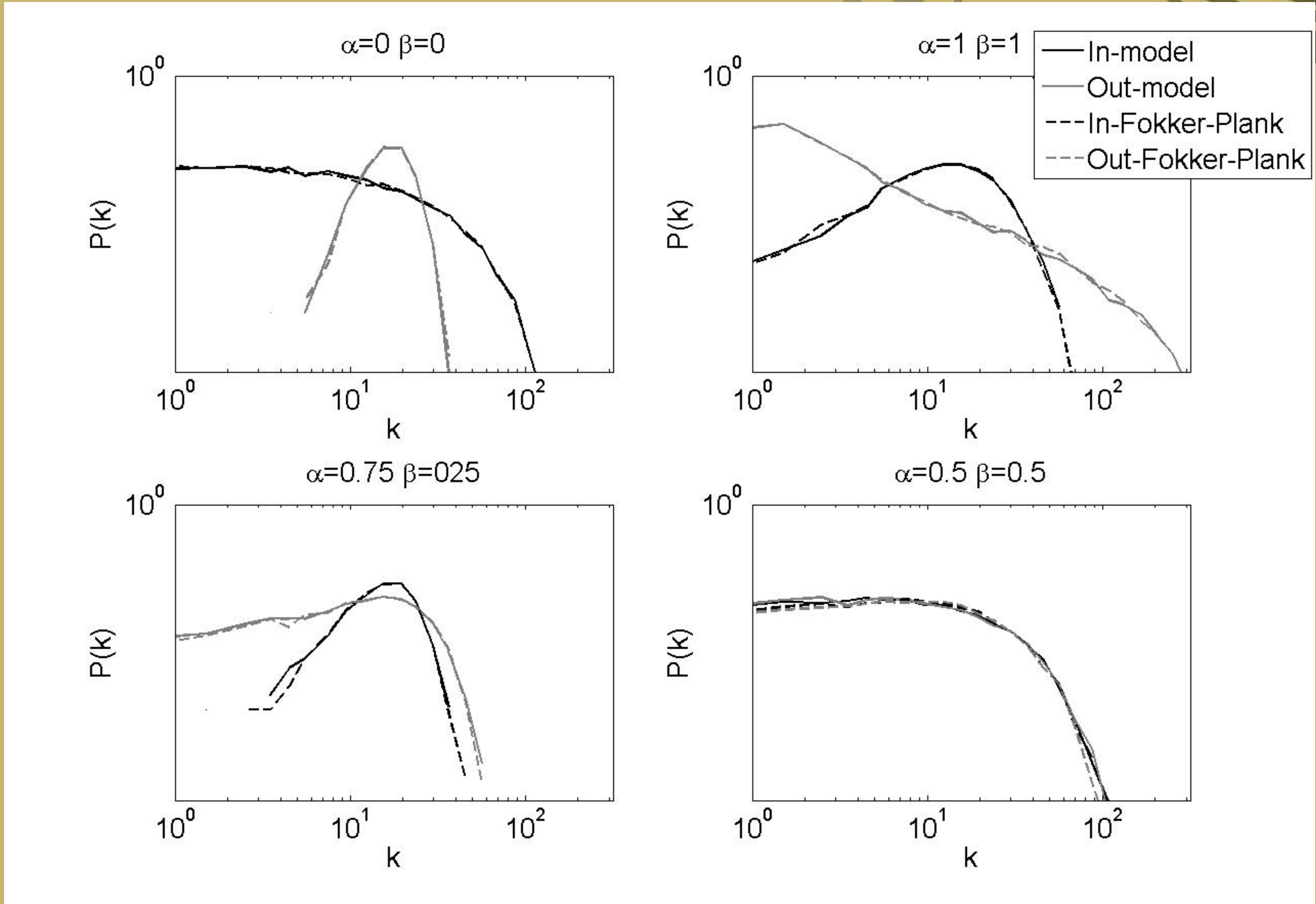
Out degree removal:

$$p(k_{out} \mid k_{in} \rightarrow k_{out}-1)=\alpha+\left(1-\alpha\right)\frac{k_{out}}{K}$$

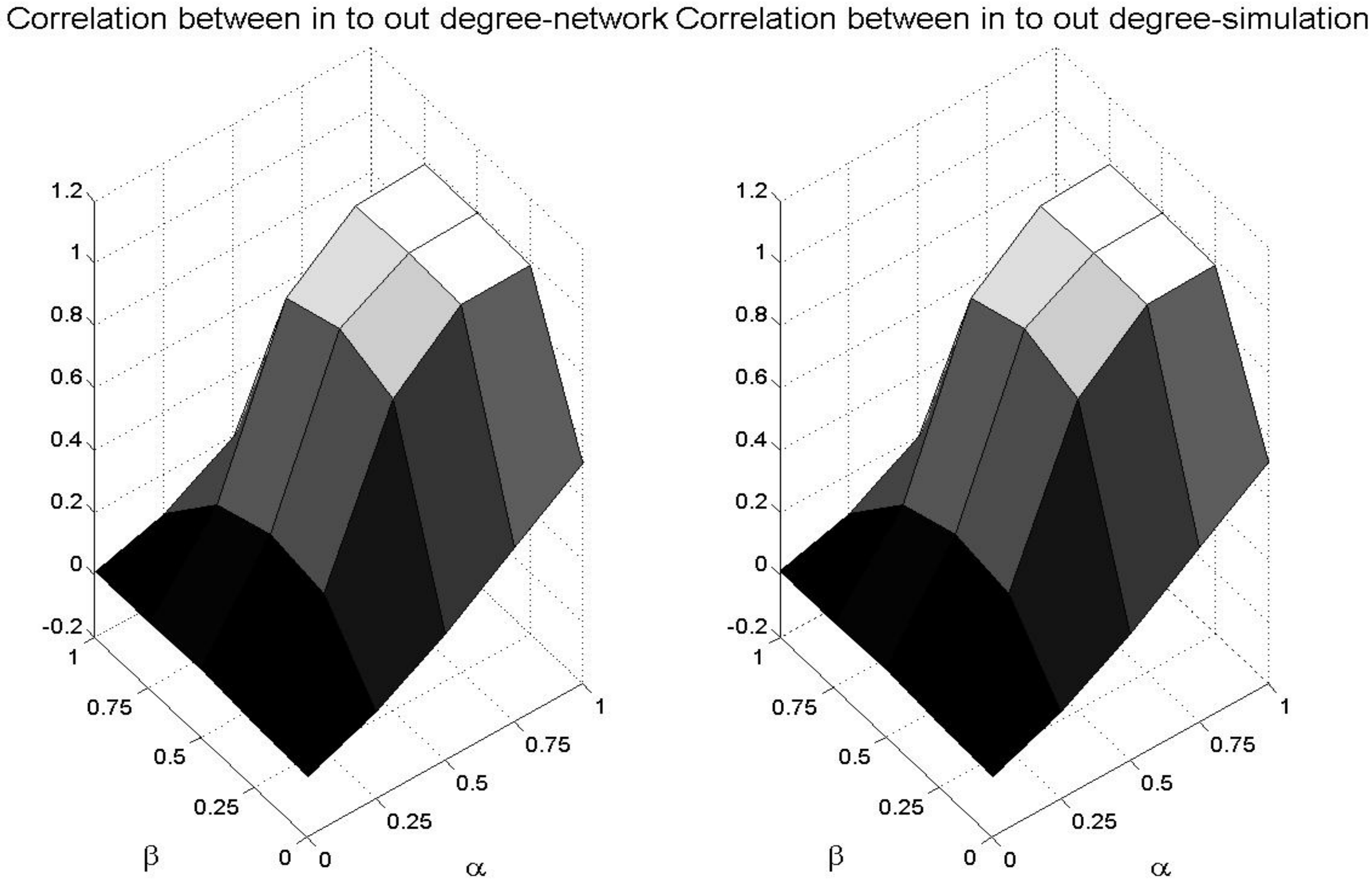
Degree distribution of model's parametric range



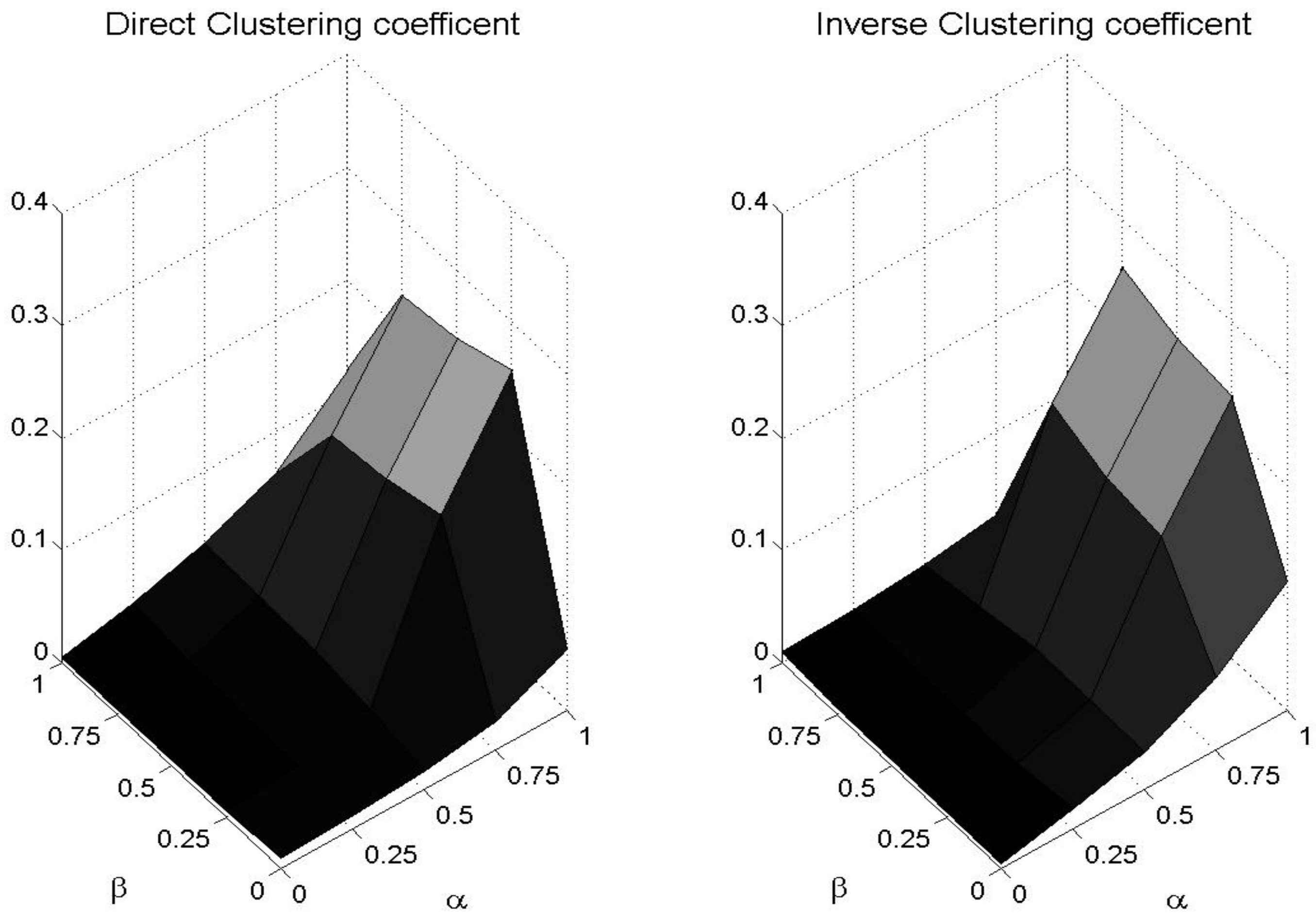
Comparison between simulation to network



Degree correlation



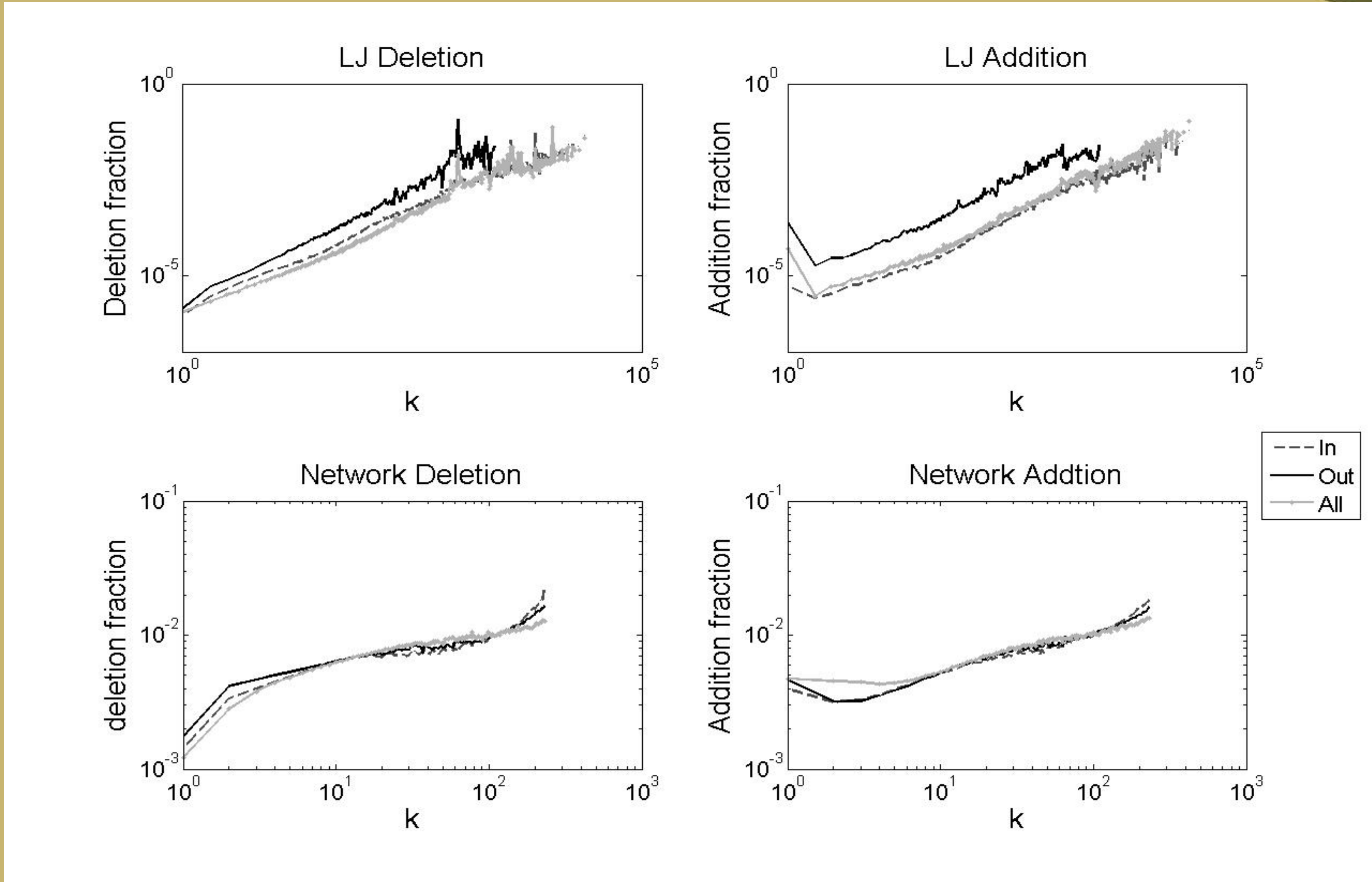
Directional clustering coefficient



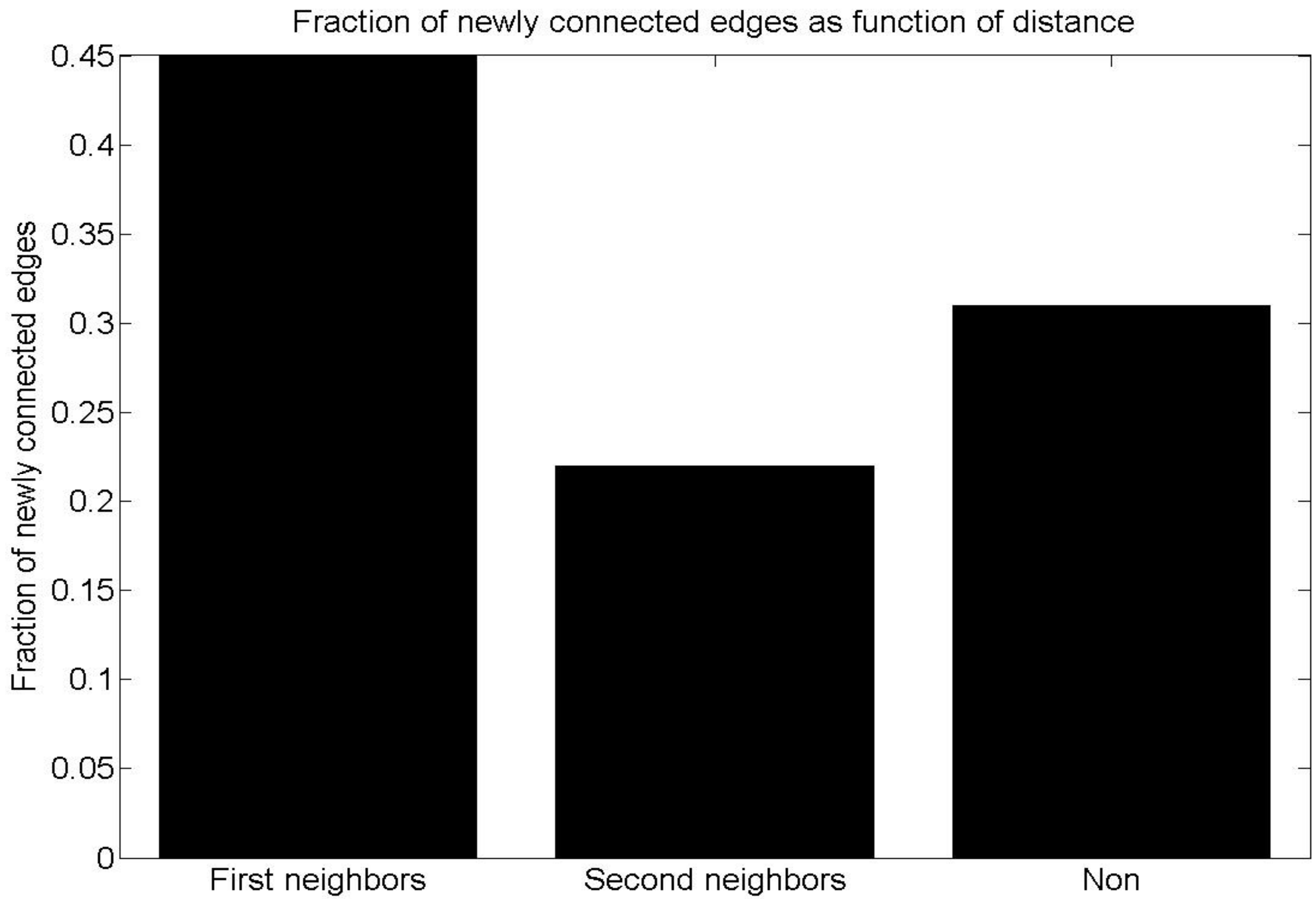
Validation of the model dynamics

- Triadic closure implies that edges are added mainly between second neighbors. However, even if , a small fraction of nodes is still connected to random nodes through the addition of random edges when no pair of yet unconnected neighbors exists.
- A preferential attachment, a process by which edges addition probability is proportional to the in/out degree of the node.
- Edges are deleted proportionally to the in/out degree of the node, as extensively discussed above.

Edge's removal and addition rate as a function of degree for model and Live Journal



Fraction of newly added edges as a function of the distance before addition



Summary

- When applied triadic closure to directed networks, it fails to explain the often observed difference between the in and out degree distribution and clustering coefficients.
- The edge deletion mechanism should be taken into account in order to properly reproduce the effect of edge direction on the degree distribution and the clustering coefficient, as well as the correlation between the in and out degrees of nodes.
- Considering network directionality, the differences between the properties of incoming and outgoing edges represent a fundamental dynamic difference. While the properties of outgoing edges are often determined by the source node, the properties of incoming edges are the cumulative results of the action of many nodes pointing to the current nodes.

Under review, European Journal of Physics B.

Thanks for you attention