

IMT INSTITUTE FOR ADVANCED STUDIES LUCCA

# MULTILEVEL COMPLEX NETWORKS AND SYSTEMS

Guido Caldarelli *IMT and LIMS, London UK* Stefano Battiston *UZH University of Zurich, Switzerland* Michelangelo Puliga *IMT Lucca, Italy* Walter Quattrociocchi *IMT Lucca, Italy* Antonio Scala *ISC-CNR Rome, Italy and LIMS UK* 

Sim



DTRA Grant HDTRA1-11-1-0048



What is a Multilevel Network ?

Actually many things.

Hopefully we are starting to classify clearly some cases Mostly they are networks of networks In this case they can be:

- multilayers of the same set of vertices (multiplex)
- the result of a coarse grained process
- they can be the result of a temporal evolution

More interestingly in the case of a set of the same vertices The inter-layer connection could be of a unknown nature

INTRODUCTION



# Multilevel Complex Networks

(overview)







Countries Network



SUMMARY

There are even more complicated situations leading to multilevel complex networks

We often do not know how to represent the interaction between two networks

One network can be created by another one or being simply related by effects of media

The first case is that of opinion dynamics, with a simple structure of interaction between users and media The second case is the network of Credit Default Swaps, a financial instrument similar to an insurance on companies default.



## **Opinion Dynamics Model**

The two networks we are considering here are

- The network of users (gossipers)
- The network of media that interacts with the first one

We take the topologies of these two networks as parameters of the problem and we checked the results in case of

- Random/complete graph
- Small world
- Scale-free (Barabási-Albert)

**MEDIA NETWORKS** 



### A simple model of opinion dynamics



(a) Gossipers interact among themselves choosing a neighbor in their social network (double arrow). (b) Gossipers are also influenced by the media: when they are exposed to information, their opinion will converge to such information if it is not too far from the gossiper's initial opinion (c) Each media chooses to mimic/oppose the most successful (the one with more followers) of its neighboring media



#### **Gossiper Network**

Gossiper Network evolves in this way Gossipers interact through the bounded confidence model (BCM)

i.e., at each step t

a gossiper *i* chooses at random a neighbor *j* in its social network and adjusts its opinion  $x_i$  according to

(a) gossip-gossip

$$\mathbf{x}_{i}^{t+1} = \mathbf{x}_{i}^{t} + \mu_{gg}(\mathbf{x}_{j}^{t} - \mathbf{x}_{i}^{t})\theta(\sigma_{gg} - |\mathbf{x}_{j}^{t} - \mathbf{x}_{i}^{t}|)$$

where

- $x_j$  is the opinion of the gossiper *j*,
- $\mu_{gg}$  is a convergence factor
- $\sigma_{gg}$  is the threshold above which gossipers do not interact.



### **Gossiper and Media**

We assume that also the interaction with the media has a similar form:

$$\boldsymbol{x}_{i}^{t+1} = \boldsymbol{x}_{i}^{t} + \boldsymbol{\mu}_{gm}(\boldsymbol{y}_{k}^{t} - \boldsymbol{x}_{i}^{t})\boldsymbol{\theta}(\boldsymbol{\sigma}_{gm} - \left|\boldsymbol{y}_{k}^{t} - \boldsymbol{x}_{i}^{t}\right|)$$

(b) gossip-media

#### where

- k is a randomly chosen media,
- $y_k$  is the information reported (meme) by the *k*-th media,
- $\mu_{gm}$  is a convergence factor
- $\sigma_{gm}$  is the threshold below which gossipers gets influenced by the media



#### Media Networks

Media are supposed to have a network of other media with which interact

either trying to copy their memes (black lines)

or trying to oppose their memes (red dashed lines).

(signed adjacency matrix J)

Each media chooses to mimic/oppose

the most successful (the one with more followers)

of its neighboring media

The number of followers is determined by

$$\boldsymbol{f}_{k}^{t} = \sum_{i} \xi_{ik}^{t} \theta \left( \boldsymbol{\sigma}_{gn} - \left| \boldsymbol{y}_{k}^{t} - \boldsymbol{x}_{i}^{t} \right| \right)$$

Where  $\xi_{ik}$  is a binary variable  $\xi_{ik}$ =1 if i chooses media k (with probability 1/m )  $\xi_{ik}$ = 0 otherwise



(c) media-media





**Results for Traditional Media Networks** 

Traditional Main Stream Media (TMSM) are few and in contact. We take them as a complete graph. Situation is qualitatively similar for more modern media



When considering only the Gossip network there is a sharp transition in opinion distances varying  $\sigma$ 

They will influence each other only if the distance between their opinions is below a given threshold (tolerance),



**Results for Traditional Media Networks** 

#### For any of the topologies considered



When adding interaction with media, opinions change and we see a smoothening in opinion distances varying  $\sigma$ 

MULTILEVEL COMPLEX NETWORKS

2 media makes a smoother transition towards homogeneity than 10

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**Results for Traditional Media Networks** 



The same results can be shown by plotting the number W of different opinions in the system

MULTILEVEL COMPLEX NETWORKS

Situation changes introducing polarization in the media



#### **Results for Polarized Media**

Then, we introduce competition (polarization) in the media dynamics: Every node of the media network, depending on the edge signature (positive or negative), can diverge (or converge) to (or from) the value of the most followed media

First, the meme of the k -th media is influenced by the most successful (the leader) I(k) of its neighbors

$$I(k) = \max_{q} \left\{ \left| J_{qk} \right| f_{q} \right\}$$

Where

 $J_{qk}$  is a signed adjacency matrix (i.e.  $J_{qk}=1$  friend,  $J_{qk}=-1$  enemy)  $f_q$  is the number of followers of media q



**Results for Polarized Media 2** 

Once we know the leader of the neighbours we update the meme according to

$$\mathbf{y}_{k}^{t+1} = \mathbf{B} \left[ \mathbf{y}_{k}^{t} + \mu_{mm} \mathbf{J}_{kl(k)} \left( \mathbf{y}_{l(k)}^{t} - \mathbf{y}_{k}^{t} \right) \boldsymbol{\theta} \left( \boldsymbol{\sigma}_{mm} - \left| \mathbf{y}_{l(k)}^{t} - \mathbf{y}_{k}^{t} \right| \right) \right]$$

Where B(y)= keeps the meme in the interval 0,1 i.e. B(y)=[1- $\theta$ (y-1)]  $\theta$ (y)y+ $\theta$ (y-1)



## **Results for Polarized Media 2**



the opinion space is maximally fragmented for both low and high values of the tolerance  $\sigma$ .

You do not have a threshold value above which you reach consesus.



#### Summary for Opinion Dynamics

People interact influencing each other opinions if the distance between them is below a given threshold  $\sigma$ 

Media interact to increase their followers, ready to shift their topics to follow more successful media

Finally, media can compete

Media coverage smoothens transitions to homogenous state But polarization introduces fragmentation in the followers opinions

Similar behaviour holds for new media (BA- Scalefree connection)

**CREDIT DEFAULT SWAPS** 



Network of Credit Default Exposure

- The Credit Default Swaps (CDS) are a financial tool created to protect companies against the risk of default (or similar credit events) occurring on companies emitting bonds or other fixed payments financial instruments.
- A CDS contract is then formed by three actors: the CDS seller, the CDS buyer and the bond issuer. An adverse credit event on the Bond issuer triggers the liquidation of the CDS from the CDS seller. In exchange to that protection the CDS buyer pays a periodic fixed amount to the CDS seller





CREDIT DEFAULT SWAPS

 Credit Default Swaps (CDS) spreads should reflect default risk of the underlying corporate debt. Actually we see that CDS spread time series did not anticipate but only followed the increasing risk of default before the financial crisis.



CREDIT DEFAULT SWAPS



### **Capitalization and CDS Price**



The average CDS price for the 176 institutions (red) and the average market capitalization (black) of the same companies.





130 The main network measures for the various methods.
(Top Left) Network measure (i.e. number of nodes);
(Top Right) Average degree; (Bottom Left) Link density;
(Bottom Right) Minimum spanning tree average path length.





Data



As the time passes the institutions tends to reduce exposure, but the price of CDS become correlated.

It would be useful to extract information from CDS to the unknown exposure network

Unfortunately we cannot see any anticipation of fragility from CDS Network....



## CONCLUSION

Some systems (Infrastructures) are easily mapped into network of networks

In economic and social systems the correlation between the various "layers" is not so easy

Nevertheless in the case of two simple financial layers we can assess the effectiveness of the role of CDS

#### IMT STUDIES LUCCA

## Activity on Multilevel Networks

#### WHAT IS "Dynamics of Multi-Level Complex Systems (DyM-CS)"

Many artificial and natural systems are characterized by a high level of differentiation in structure and organization; they exist in areas as diverse as the Internet, energy management, climate, financial markets, infrastructures (including ICT), biology, transport, epidemics, meteorology, urban planning, social simulation and policy impact assessment. In order to describe and control these systems there is a need to observe and reconstruct their dynamics and make sense of large amounts of heterogeneous data gathered on various scales. Most of these areas would benefit from an international effort in collecting and sharing data, models and from looking for a general, common theoretical approach. The science of complex systems (CSS) offers a framework for this theoretical approach.

The original call Description - FP7: FET Proactive Intiative: Dynamics of Multi-Level Complex Systems (DyM-CS), can be found here

#### http://cordis.europa.eu/fp7/ict/fet-proactive/dymcs\_en.html



#### www.dym-cs.eu







#### MULTIPLEX NETWORKS

#### **Networks of Networks**



www.multiplexproject.eu





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INTRODUCTION



## **MULTIPLEX** project vision

Real world systems show a large number of **interdependencies**: physical interdependency; cyber interdependency; geographic interdependency; logical interdependency that need financial and political coordination.

We need to move beyond topological characterization, and understand how to **characterize**, **observe** and **control** the **dynamics** of **real** networks.



	During the project	Long term
Paradigm shift	<ul> <li>Develop data driven models that go beyond the methodological and disciplinary boundaries of a specific approach.</li> <li>Theoretical, algorithmic, and computational framework that will enable us to evaluate the onset of tipping points, emergent phenomena, cooperative phenomena in multilevel networks.</li> <li>Use the tools of control theory to go beyond network characterization and understand the control of real networks.</li> </ul>	Breakthrough in the application of complex systems, algorithmic and network theory to the modeling and analysis of the integration of social interactions and technological and communication networks Learn to measure (observe), quantify, predict, and control complex systems.





ECCS'14 will be a major international conference and event in the area of complex systems and interdisciplinary science in general. It will offer unique opportunities to study novel scientific approaches in a multitude of application areas.

The conference will cover a broad range of subjects on all aspects of Complex Systems, as reflected by the following conference tracks:

Sponsor

