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Higher-Order Aggregate Representations of Temporal Networks

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ETH Zürich

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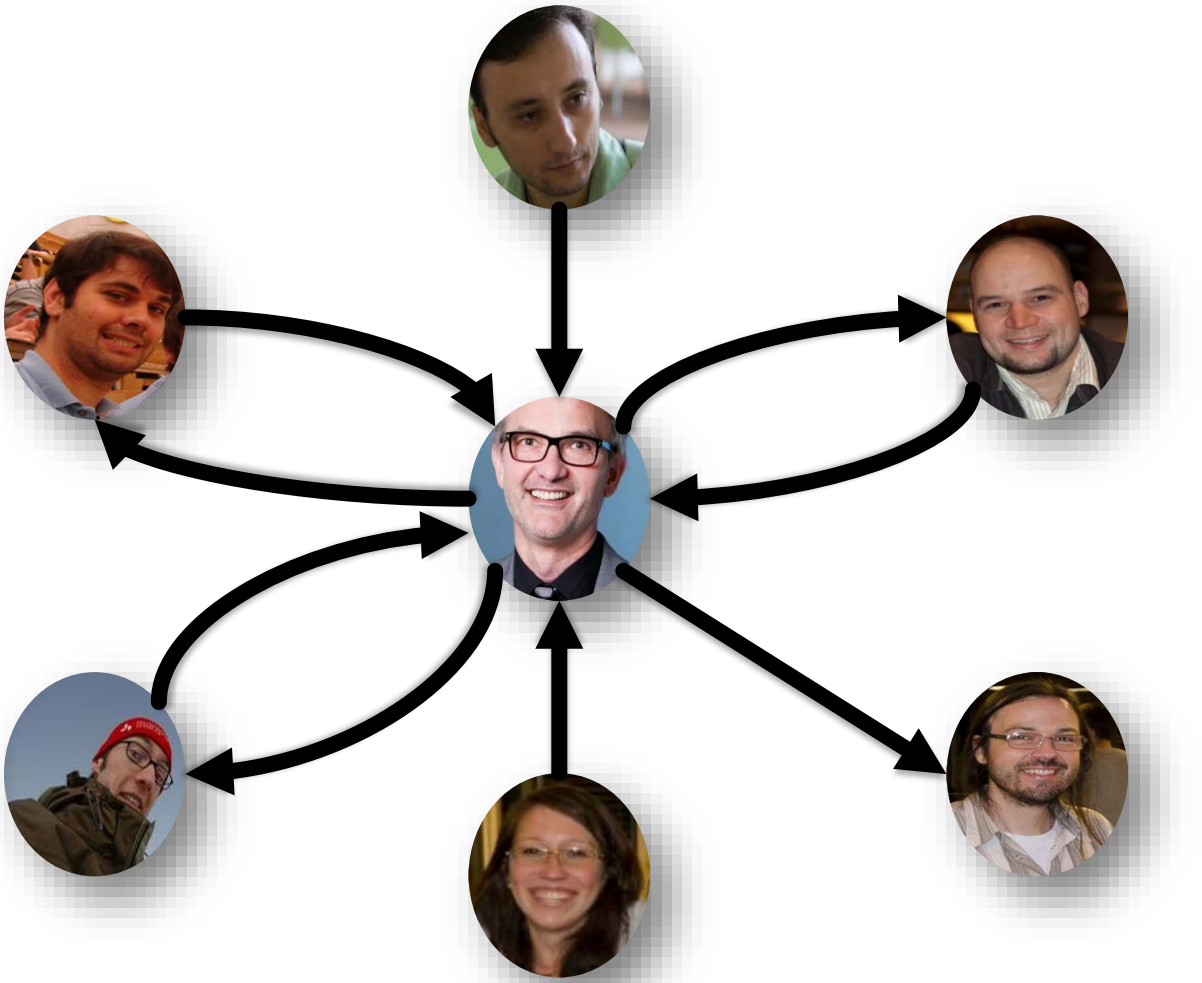
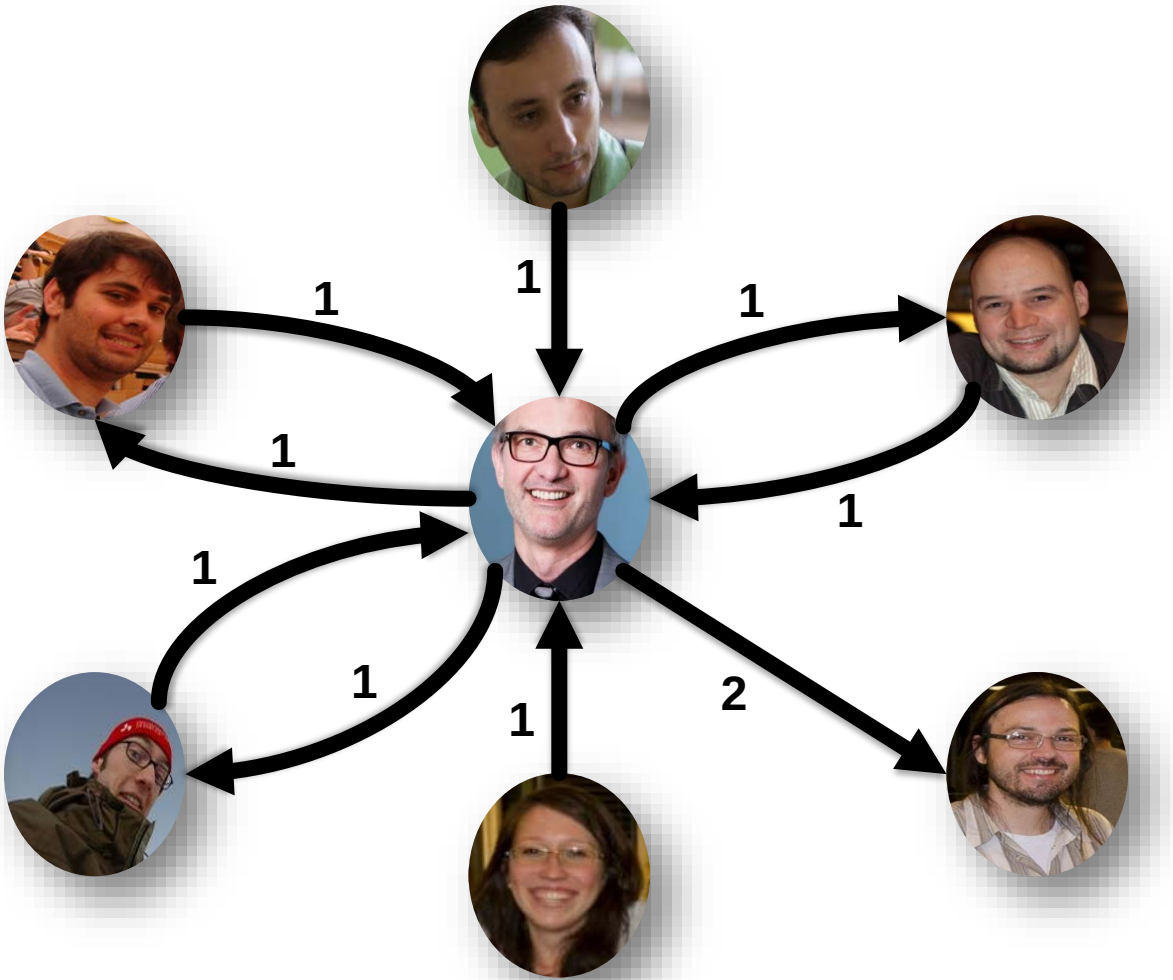
Chair of Systems Design @ ETH Zürich

data-driven modeling of complex systems

Temporal Dynamics

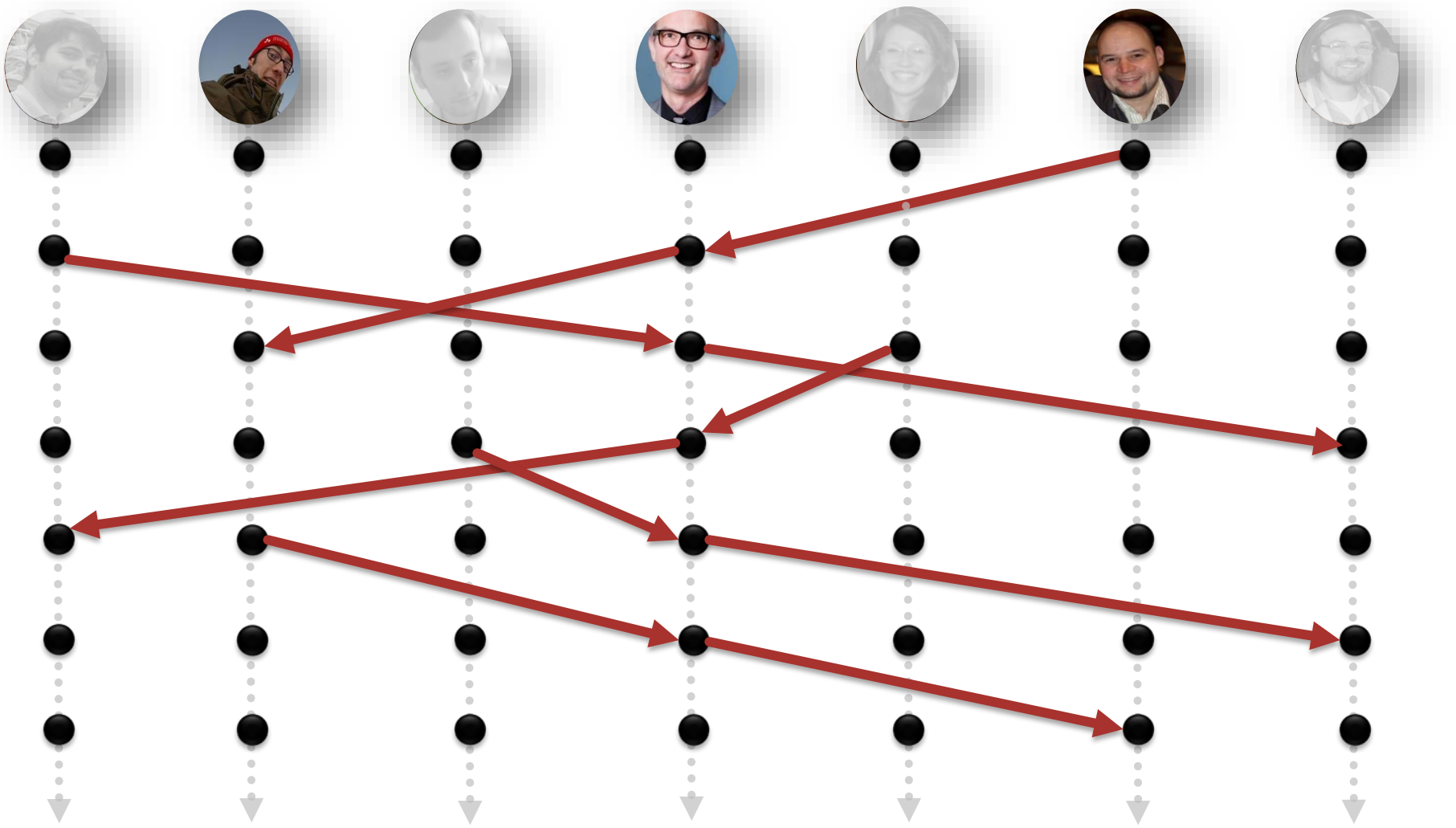


Aggregate networks

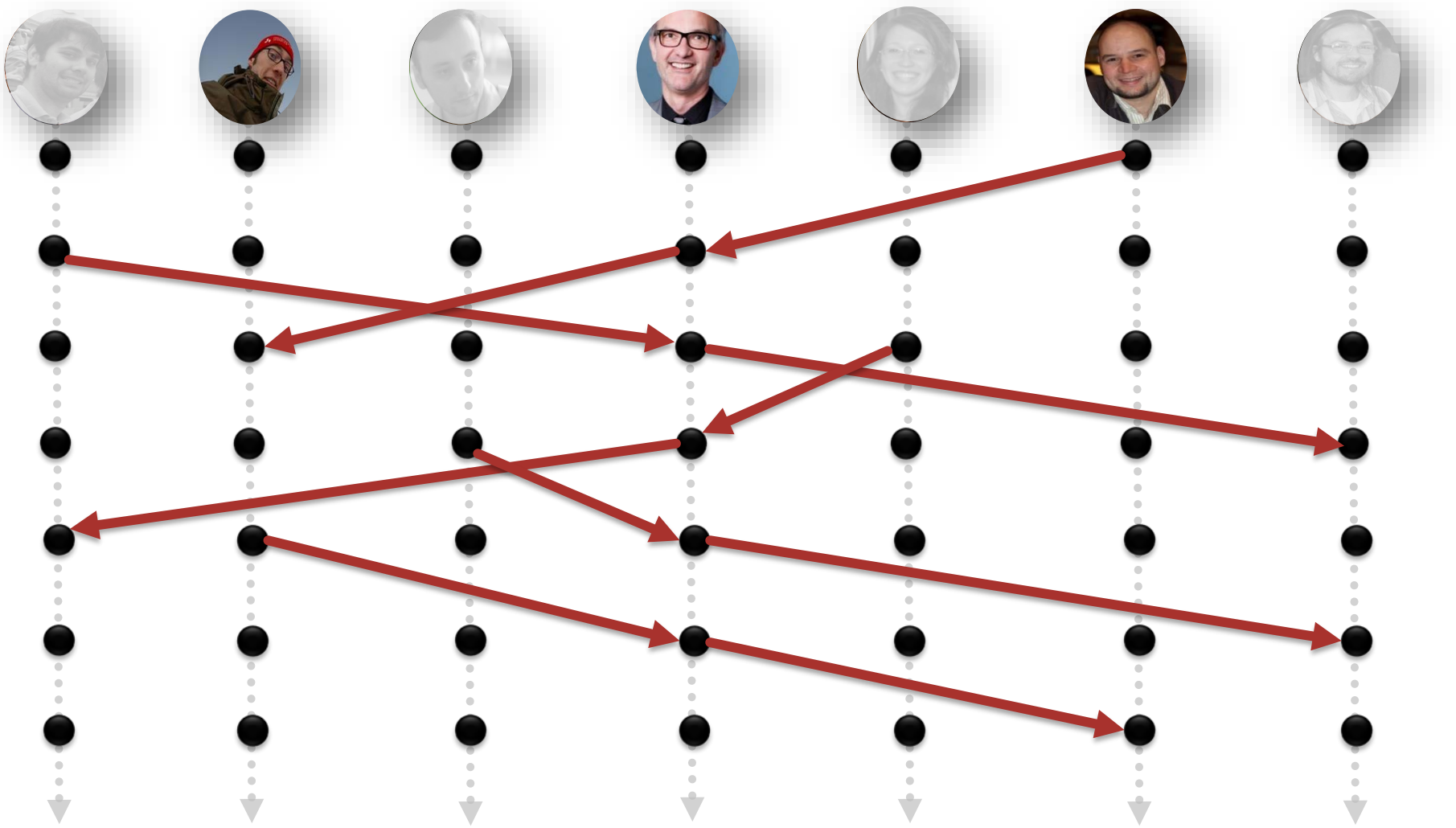


- we lose all information on the **timing of interactions**
- many studies have considered **when** nodes are active
- fewer works have studied the **ordering** of interactions
- ordering of interactions is crucial in temporal networks

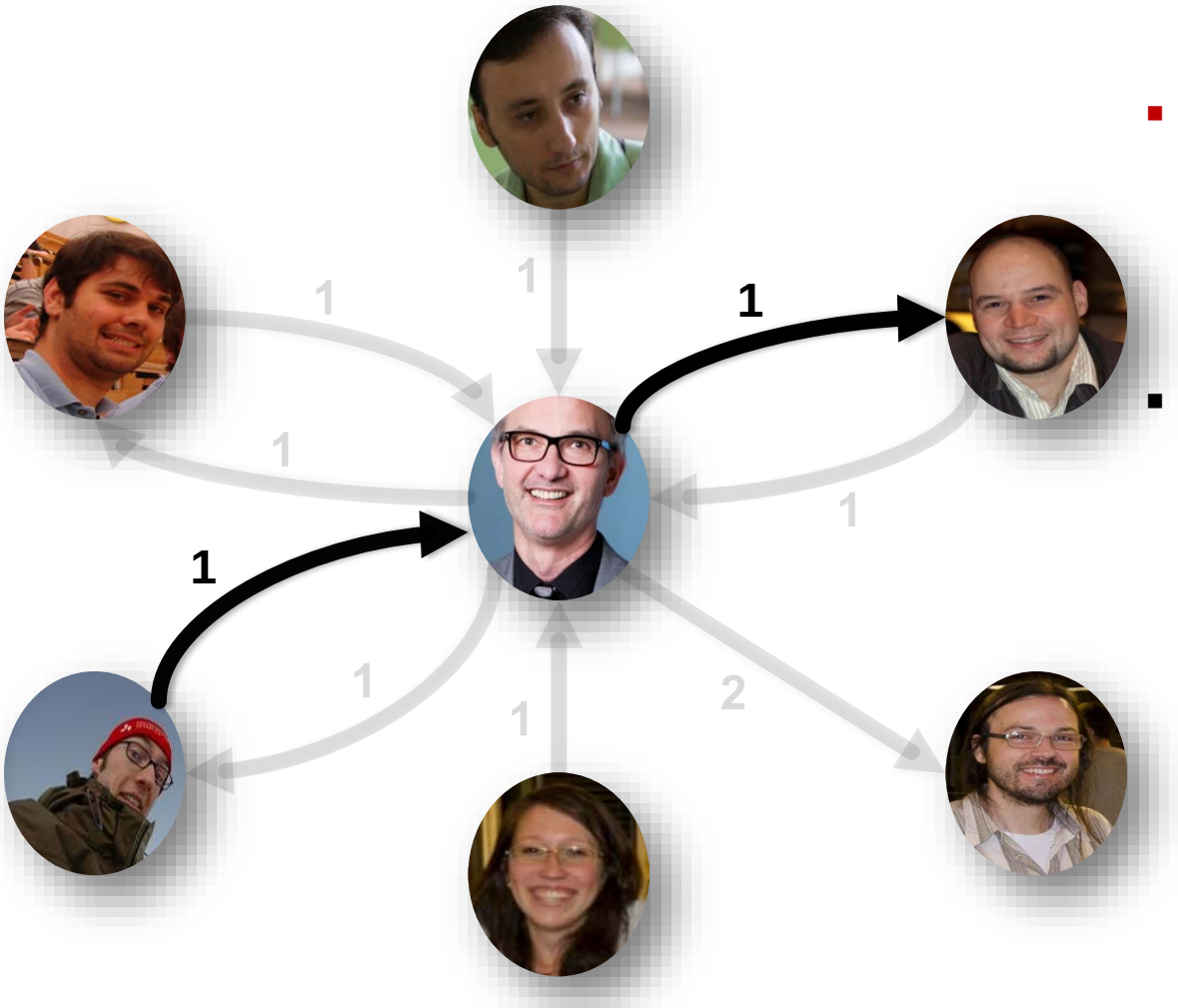
Time-respecting paths



Let‘s change the order …



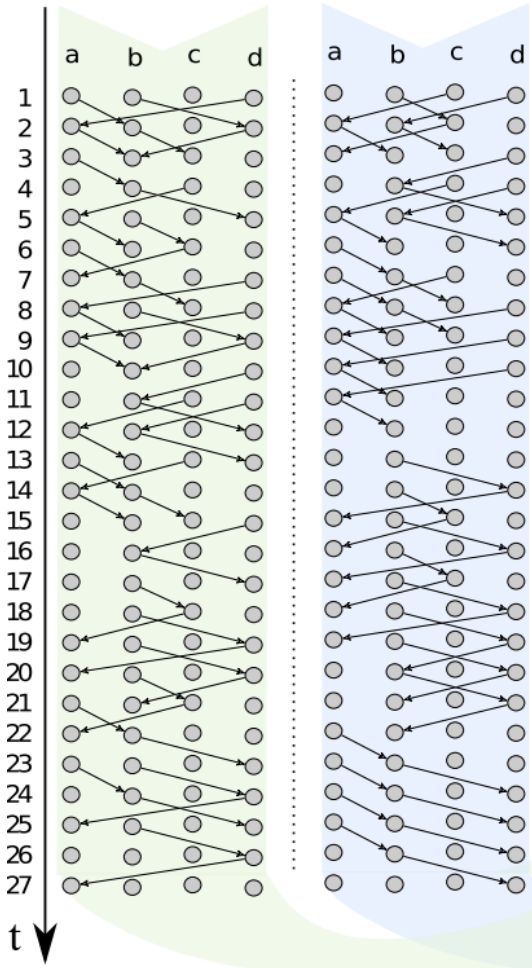
The transitivity fallacy

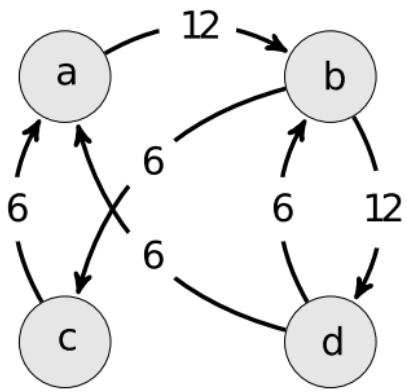


- **actual time-respecting paths are more constrained than expected** from aggregate network
- effect has been shown in
  [Pfitzner et al. 2013]
  [Lentz et al. 2013]
  [Rosvall et al. 2013]
  [Scholtes et al. 2013]
  [Lambiotte et al. 2014]

R Pfitzner, I Scholtes, A Garas, CJ Tessone, F Schweitzer: **Betweenness Preference: Quantifying Correlations in the Topological Dynamics of Temporal Networks**, Phys. Rev. Let., Vol. 110, 198701, May 10 2013

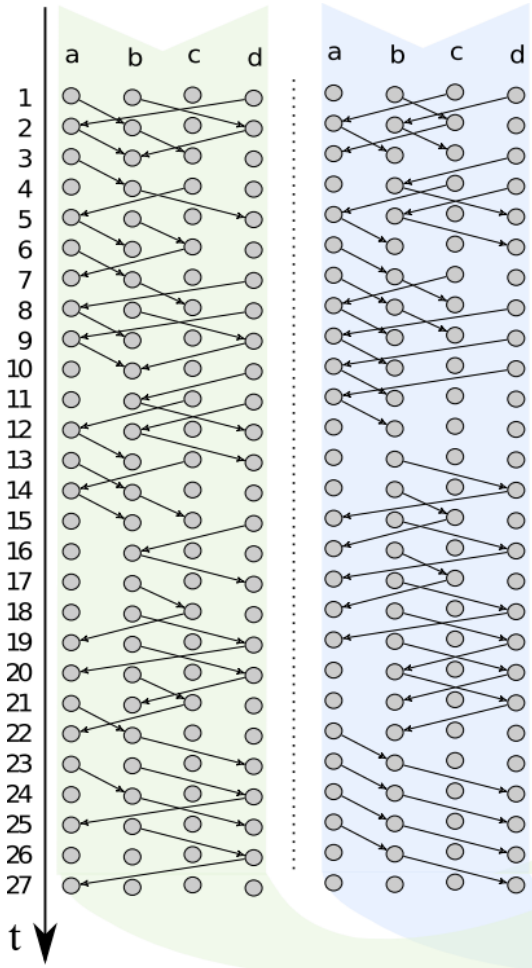
Time-aggregated networks

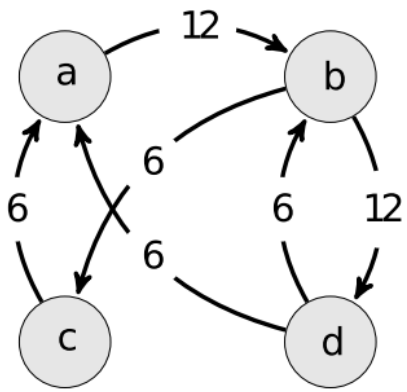




- **weights** in aggregate networks capture **statistics of edges** in temporal network
- bad model if interaction sequences deviate from **Markovian asumption**
- we **lose information on the ordering** of interactions
- **higher-order model that incorporates ordering** of interactions?

Higher-order aggregate networks?

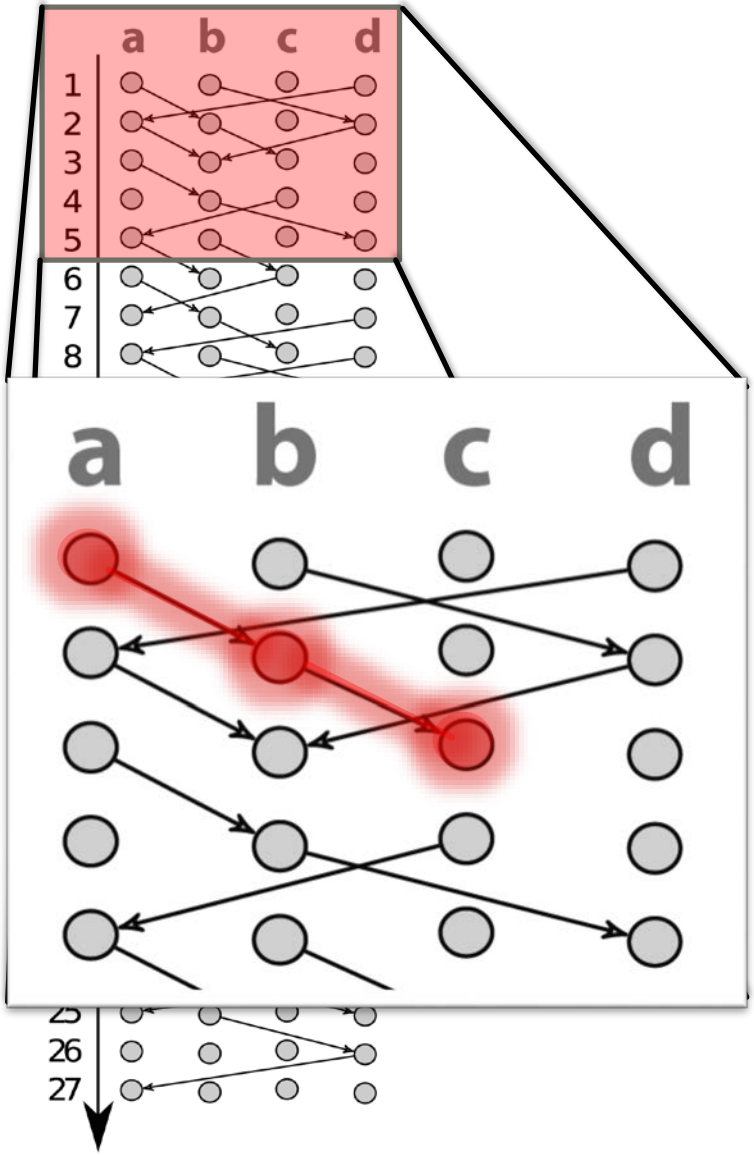


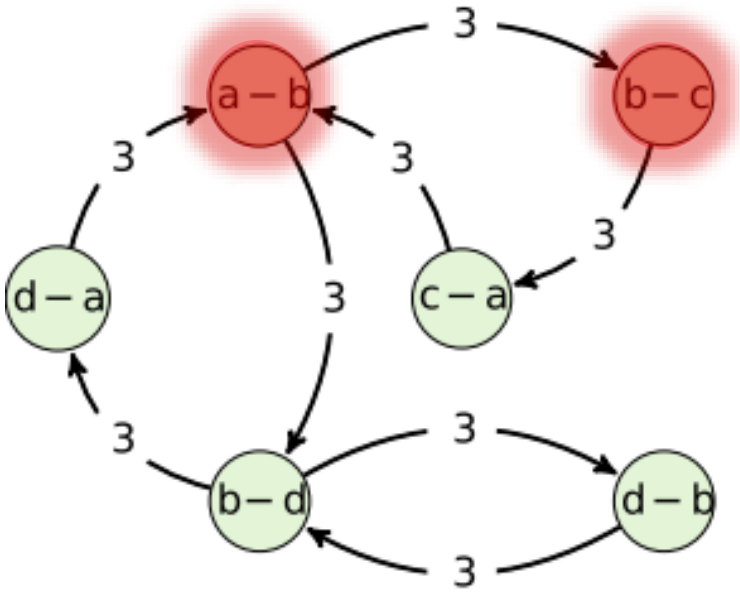


- edge weights can be viewed as statistics of **„time-respecting“ path of length one**
- time-aggregated networks are a **first-order static representation** of temporal networks
- **idea:** construct **higher-order static representations** that capture statistics of longer time-respecting paths
- what can we learn from this perspective?

I Scholtes, N Wider, R Pfitzner, A Garas, CJ Tessone, F Schweitzer: **Slow-Down vs. Speed-Up of Information Diffusion in Non-Markovian Temporal Networks,** arXiv:1307.4030, July 15 2013

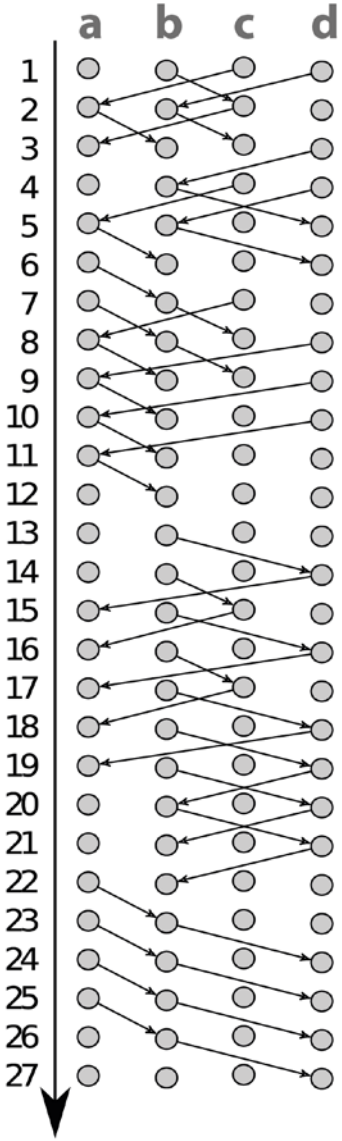
Second-order aggregate networks

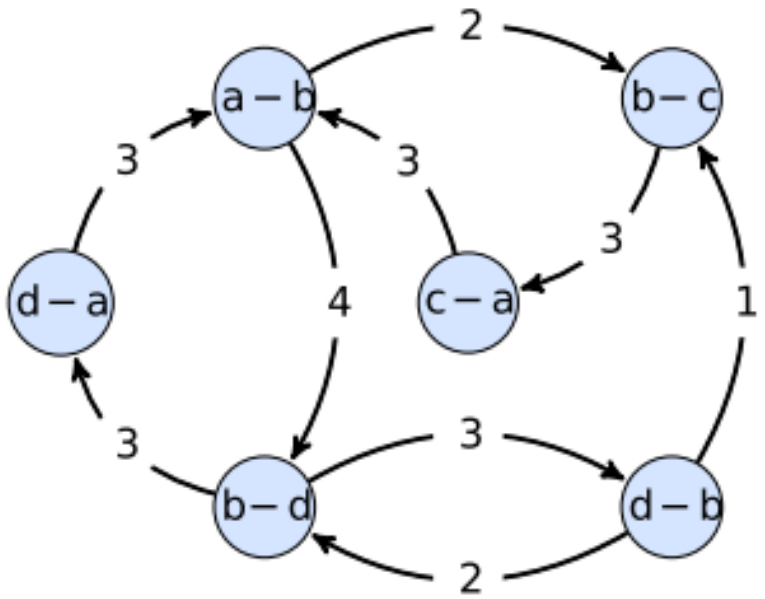




- **state space expansion:** nodes represent links in temporal network
- link (e_1, e_2) implies: e_1 **before** e_2
- weights capture statistics of **time-respecting path of length two**

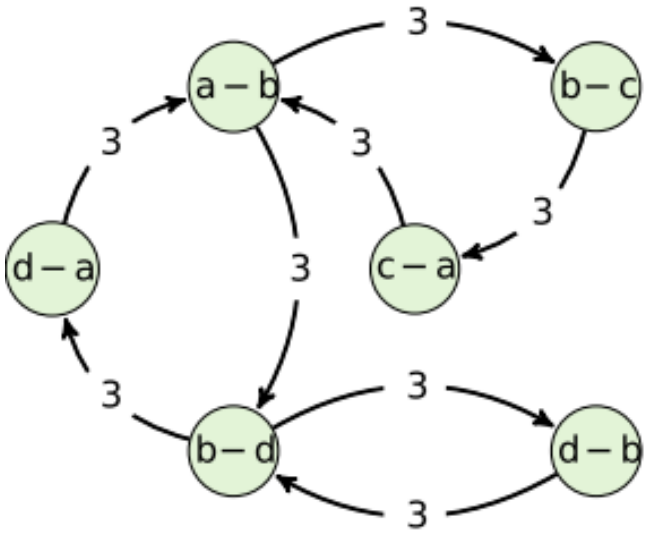
Let‘s reorder again …

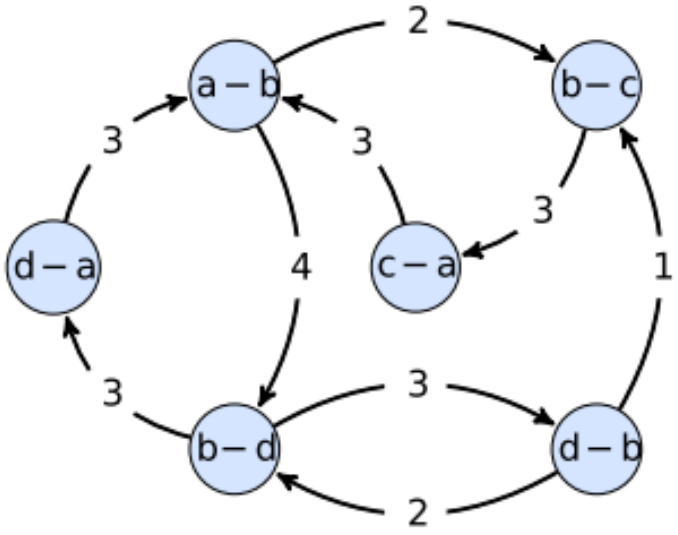




- second-order aggregate networks are **sensitive to ordering of interactions**
- simplest possible **causality-preserving static representation** of a temporal network
- we obtain the **„causal topology“** of dynamic complex systems

Causal topology of dynamic complex systems

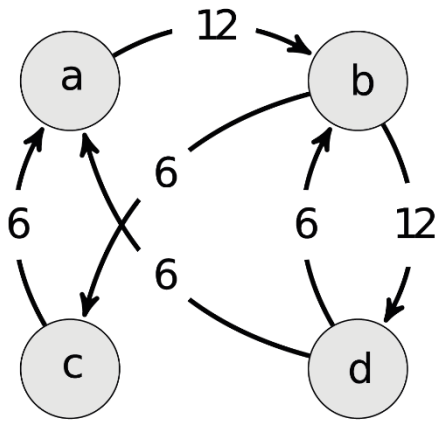




	$a-b$	$b-c$	$b-d$	$c-a$	$d-a$	$d-b$
$a-b$	0	0	0	3	3	0
$b-c$	3	0	0	0	0	0
$b-d$	3	0	0	0	0	3
$c-a$	0	3	0	0	0	0
$d-a$	0	0	3	0	0	0
$d-b$	0	0	3	0	0	0

	$a-b$	$b-c$	$b-d$	$c-a$	$d-a$	$d-b$
$a-b$	0	0	0	3	3	0
$b-c$	2	0	0	0	0	1
$b-d$	4	0	0	0	0	2
$c-a$	0	3	0	0	0	0
$d-a$	0	0	3	0	0	0
$d-b$	0	0	3	0	0	0

Higher-order Markov models

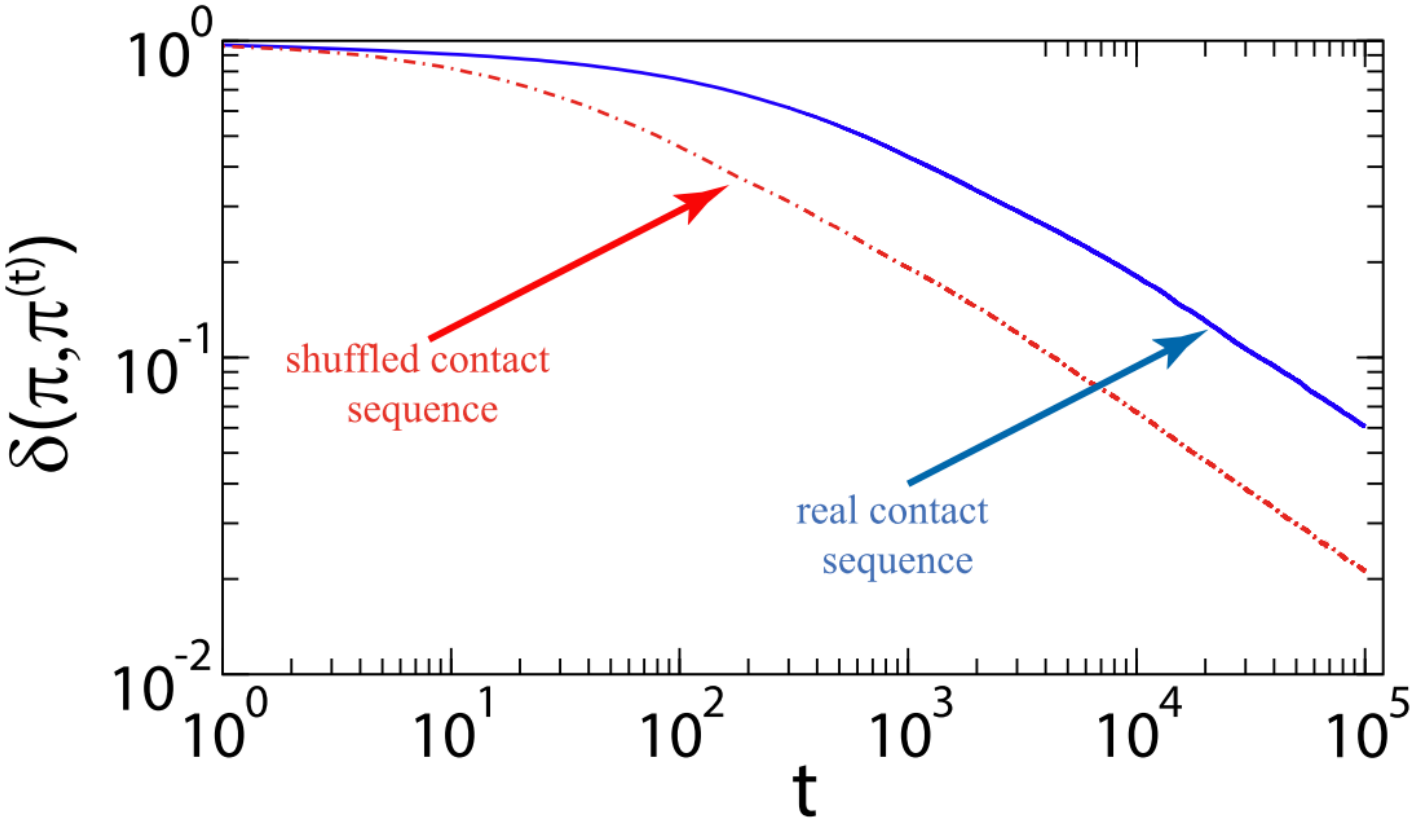


leading eigenvector = $(\frac{1}{4}, \frac{1}{8}, \frac{1}{4}, \frac{1}{8}, \frac{1}{8}, \frac{1}{8}, \frac{1}{8})$

	$a-b$	$b-c$	$b-d$	$c-a$	$d-a$	$d-b$
$a-b$	0	0	0	1	1	0
$b-c$	$\frac{1}{2}$	0	0	0	0	0
$b-d$	$\frac{1}{2}$	0	0	0	0	1
$c-a$	0	1	0	0	0	0
$d-a$	0	0	$\frac{1}{2}$	0	0	0
$d-b$	0	0	$\frac{1}{2}$	0	0	0

	$a-b$	$b-c$	$b-d$	$c-a$	$d-a$	$d-b$
$a-b$	0	0	0	1	1	0
$b-c$	$\frac{1}{3}$	0	0	0	0	$\frac{1}{3}$
$b-d$	$\frac{2}{3}$	0	0	0	0	$\frac{2}{3}$
$c-a$	0	1	0	0	0	0
$d-a$	0	0	$\frac{1}{2}$	0	0	0
$d-b$	0	0	$\frac{1}{2}$	0	0	0

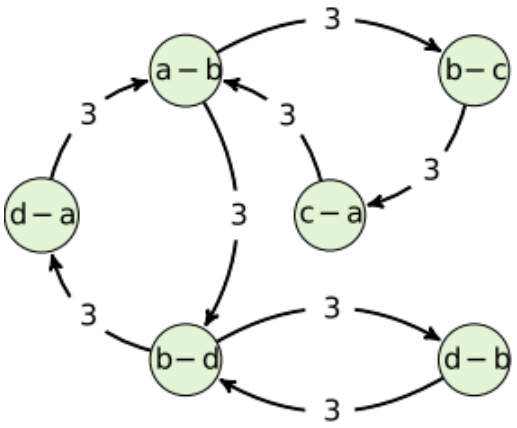
Application 1: Diffusion in temporal networks

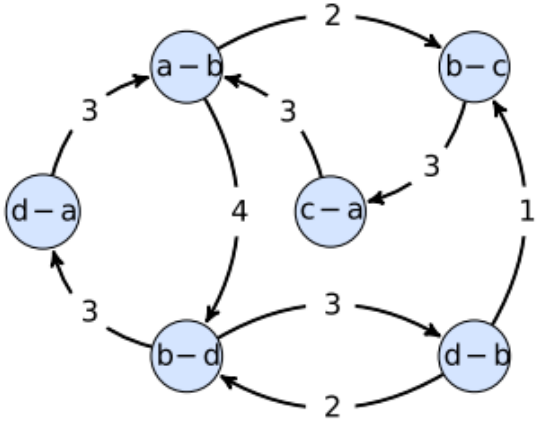


random walk convergence in **empirical** and **shuffled** temporal network extracted from interactions in an ant colony

I Scholtes, N Wider, R Pfitzner, A Garas, CJ Tessone, F Schweitzer: **Slow-Down vs. Speed-Up of Information Diffusion in Non-Markovian Temporal Networks,** arXiv:1307.4030, July 15 2013

Spectral properties of higher-order models





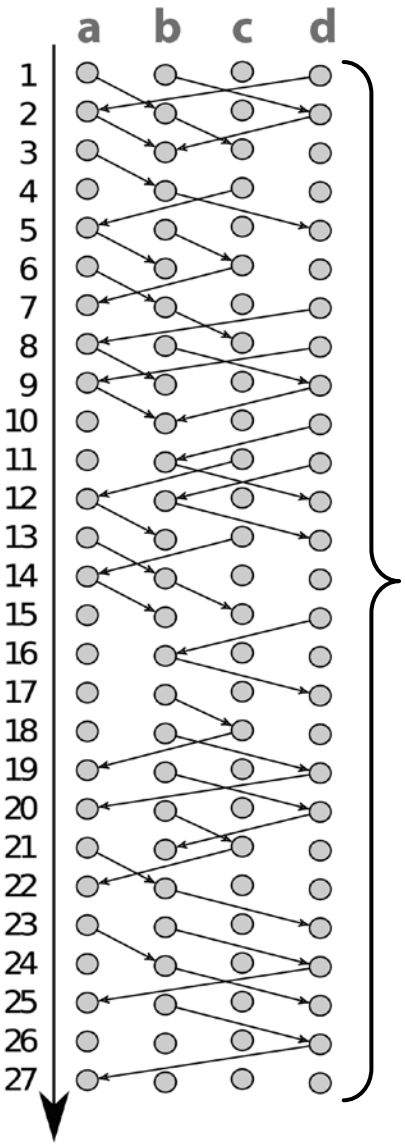
$$|\lambda_2| = 0.872$$

$$|\lambda_2| = 0.816$$

	$a-b$	$b-c$	$b-d$	$c-a$	$d-a$	$d-b$
$a-b$	0	0	0	1	1	0
$b-c$	$\frac{1}{3}$	0	0	0	0	$\frac{1}{3}$
$b-d$	$\frac{2}{3}$	0	0	0	0	$\frac{2}{3}$
$c-a$	0	1	0	0	0	0
$d-a$	0	0	$\frac{1}{2}$	0	0	0
$d-b$	0	0	$\frac{1}{2}$	0	0	0

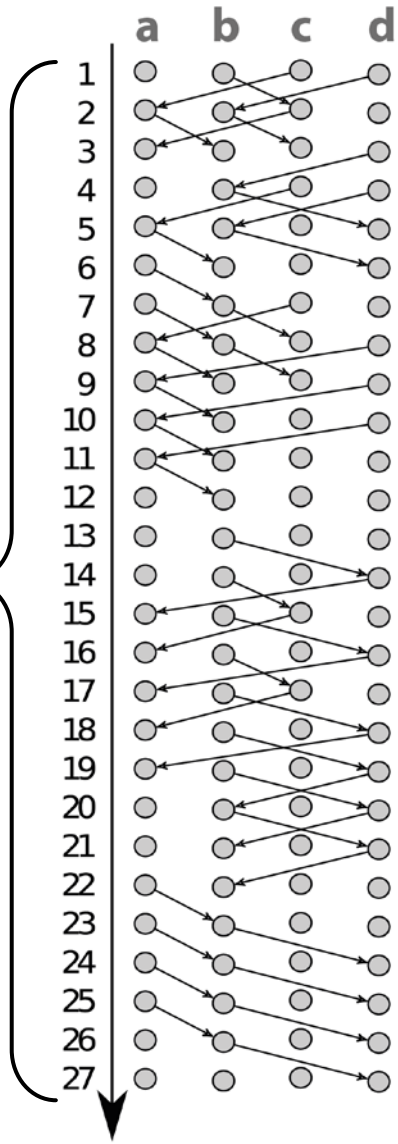
	$a-b$	$b-c$	$b-d$	$c-a$	$d-a$	$d-b$
$a-b$	0	0	0	1	1	0
$b-c$	$\frac{1}{2}$	0	0	0	0	0
$b-d$	$\frac{1}{2}$	0	0	0	0	1
$c-a$	0	1	0	0	0	0
$d-a$	0	0	$\frac{1}{2}$	0	0	0
$d-b$	0	0	$\frac{1}{2}$	0	0	0

Spectral gap of temporal networks

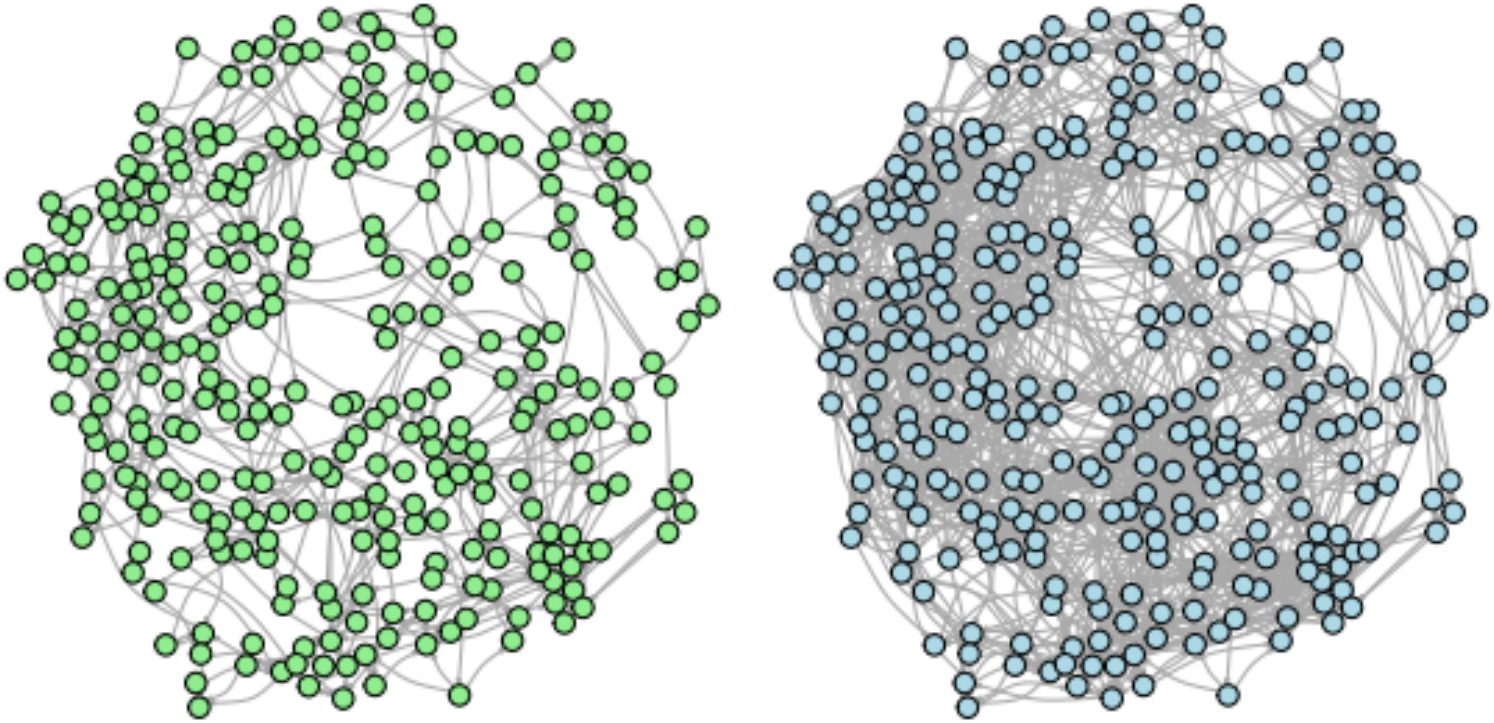


$|\lambda_2| = 0.872$

$|\lambda_2| = 0.816$



Causal topology of ant colony



causal topology of **empirical** and **shuffled** temporal network extracted from interactions in an ant colony

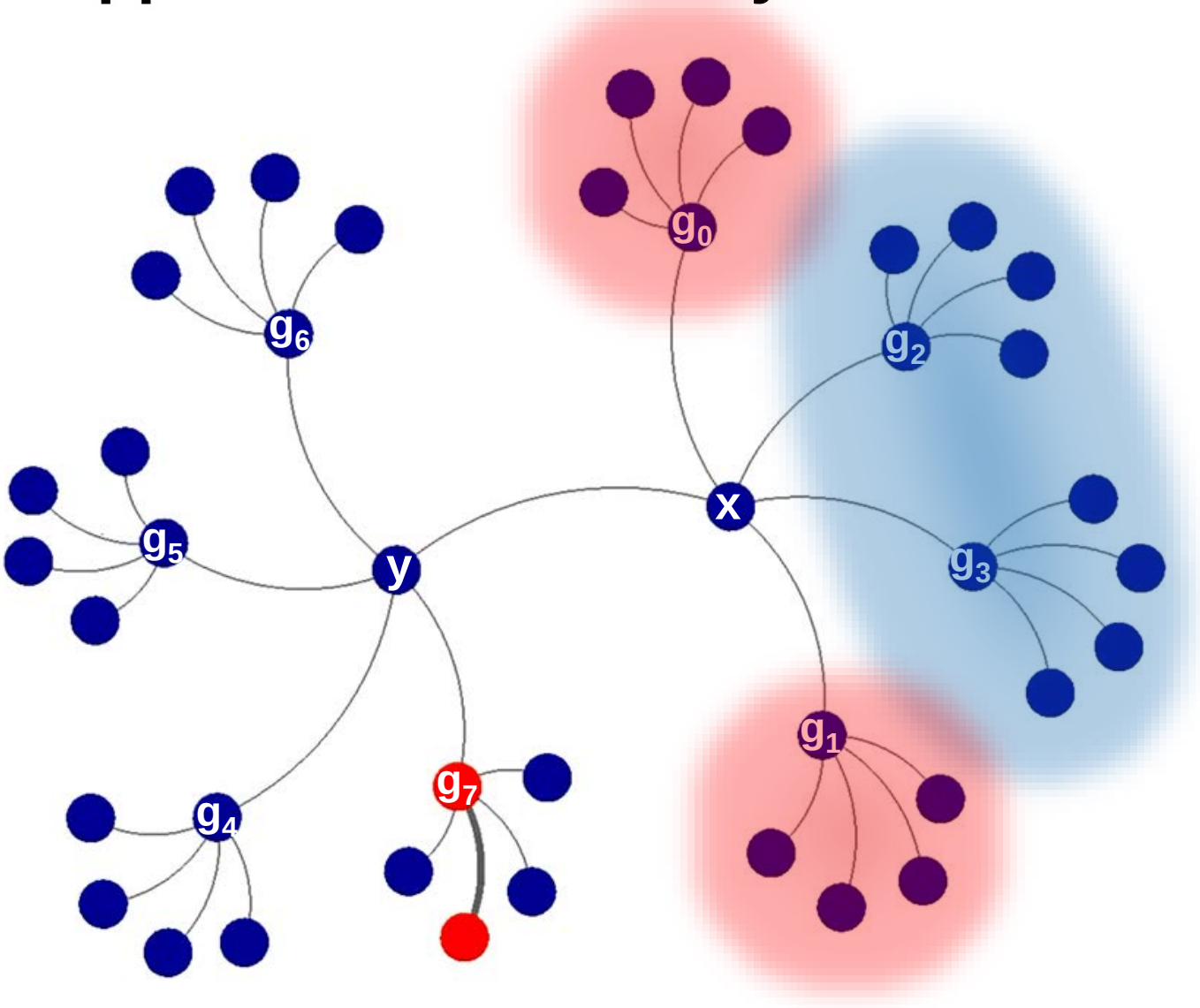
ratio of spectral gaps accurately predicts slow-down of diffusion in empirical temporal network

I Scholtes, N Wider, R Pfitzner, A Garas, CJ Tessone, F Schweitzer: **Slow-Down vs. Speed-Up of Information Diffusion in Non-Markovian Temporal Networks,** arXiv:1307.4030, July 15 2013

Speed-up of diffusion?

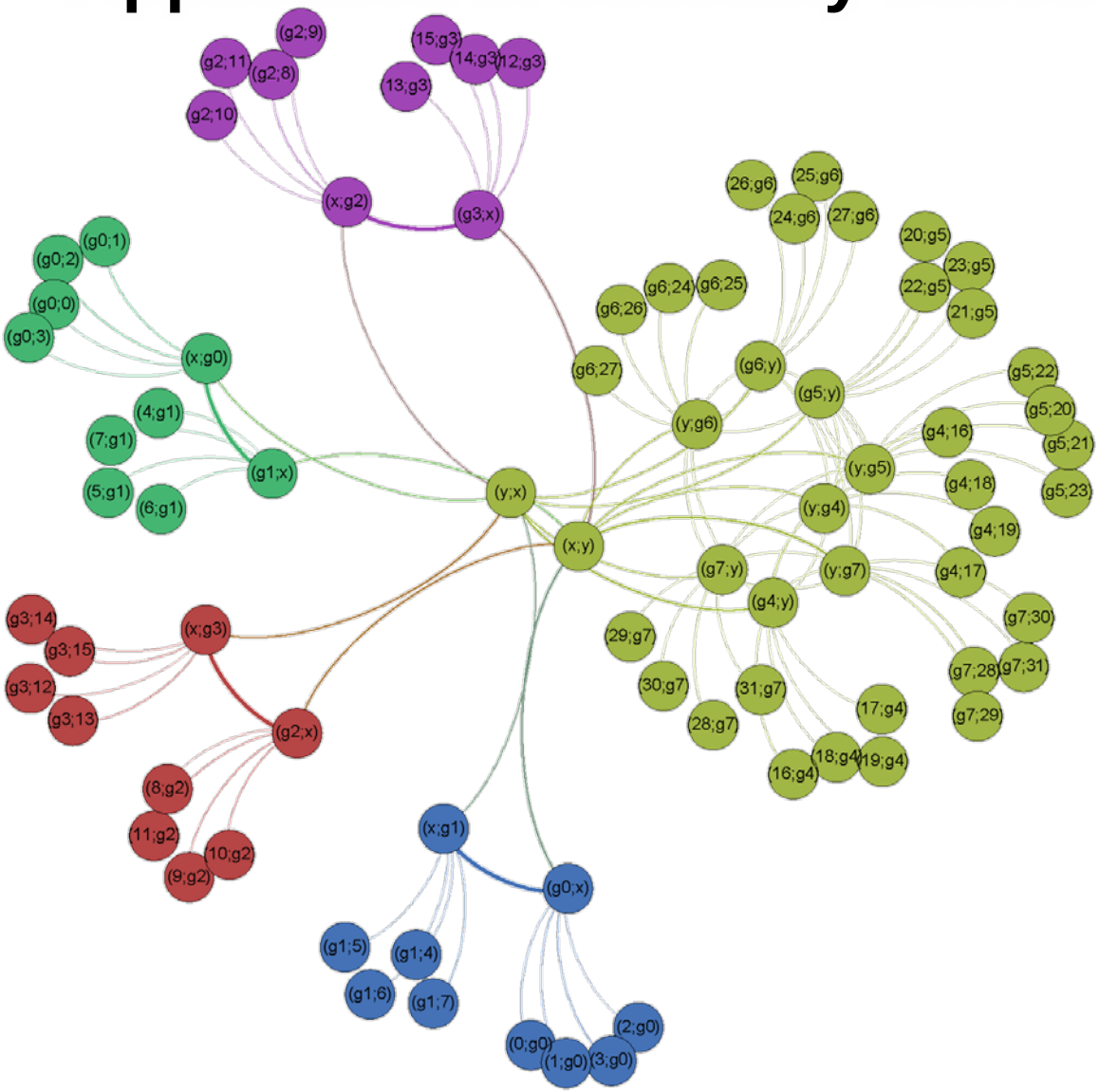
Learn more on Friday 3:40 pm!

Application 2: causality-based ranking



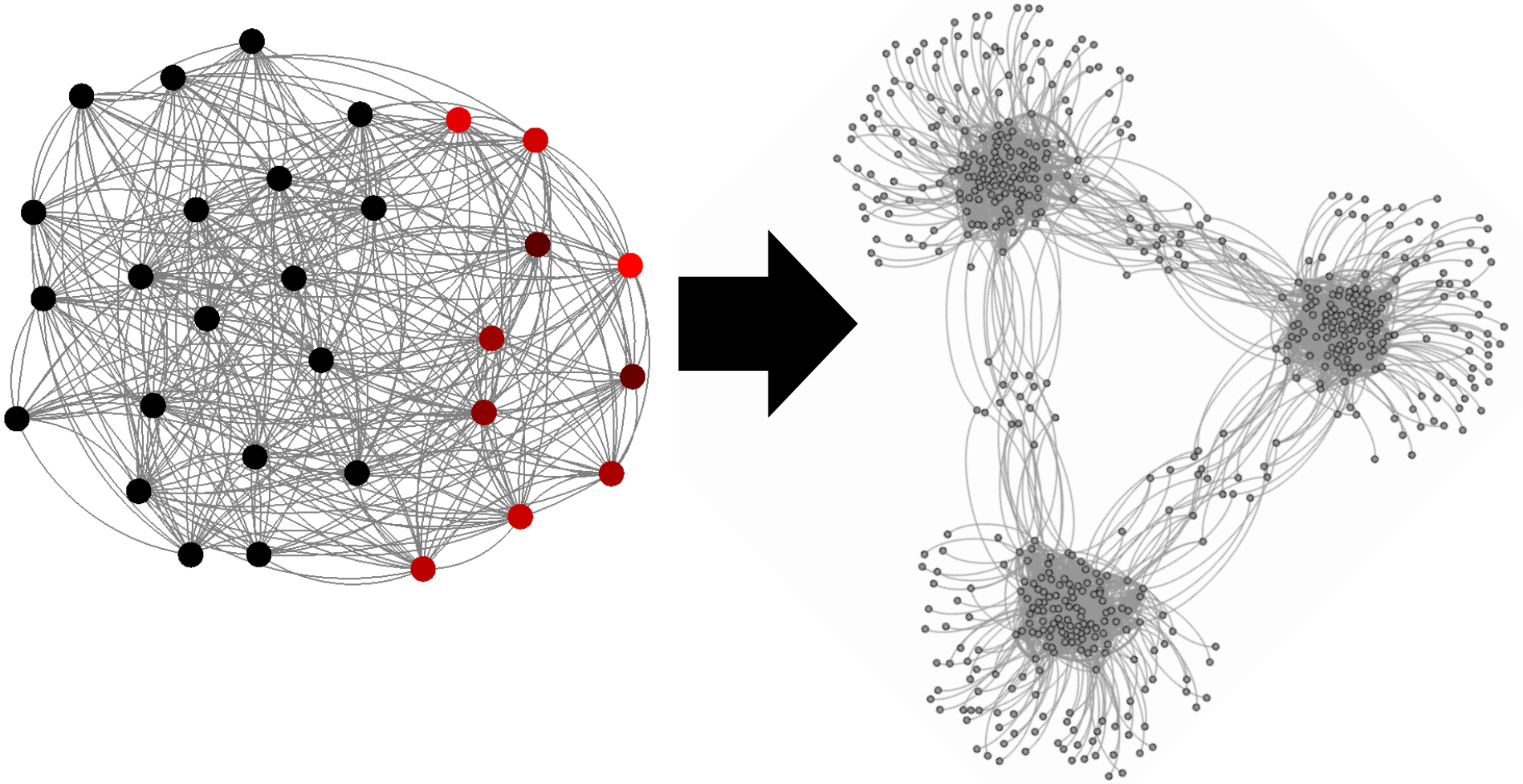
rank	node
#1	x
#1	y
#2	g0
#2	g1
...	
#2	g7
#3	0
...	...
#3	31

Application 2: causality-based ranking

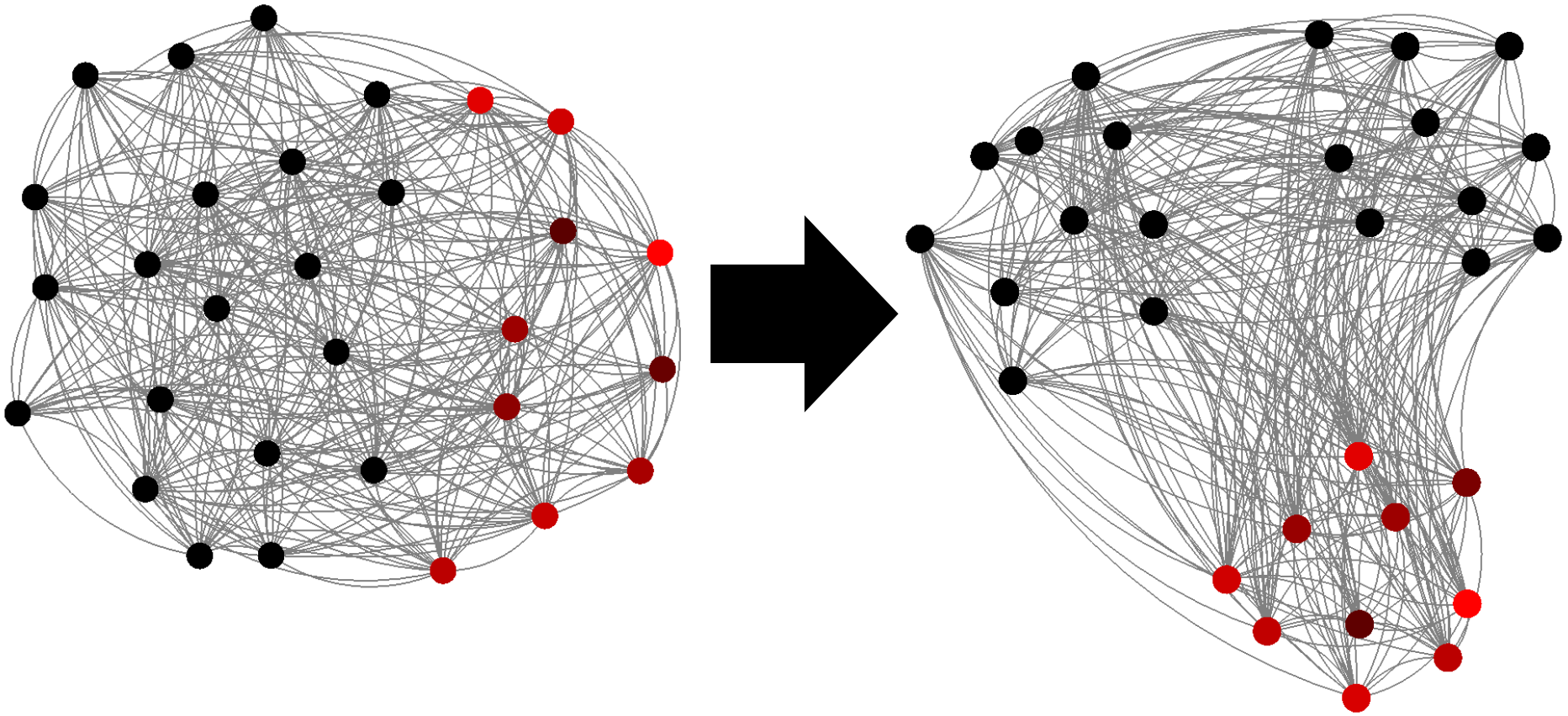


rank	node
#1	y
#2	x
#3	g7
#3	g6
#3	g5
#3	g4
#4	g3
#4	g2
#4	g1
#4	g0
#5	0
...	
#5	31

Application 3: temporal community detection



Application 4: network visualisation



I Scholtes et al.: **Force-Directed Layout of Non-Markovian Temporal Networks**, working paper, draft available online, May 06 2014

Thank you!

R Pfitzner, I Scholtes, A Garas, CJ Tessone, F Schweitzer: **Betweenness Preference: Quantifying Correlations in the Topological Dynamics of Temporal Networks**, Physical Review Letters, Vol. 110, 198701, May 10 2013

I Scholtes, N Wider, R Pfitzner, A Garas, CJ Tessone, F Schweitzer: **Slow-down vs. Speed-up of Information Diffusion in Non-Markovian Temporal Networks**, arXiv:1307.4030, July 15 2013

I Scholtes et al.: **Force-Directed Layout of Non-Markovian Temporal Networks**, working paper, available online at http://www.sg.ethz.ch, May 06 2014

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